TMUA9. Model Answer.

Cambridge Assessment Admissions Testing

TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION **D513/12**

PAPER 2

Wednesday 8 November 2017 Time: 75 minutes

Additional Materials: Answer sheet

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one.
You also require a soft pencil and an eraser.

This paper is the second of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

Any rough work should be done on this question paper. No extra paper is allowed.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

Calculators and dictionaries must **NOT** be used.
There is no formulae booklet for this test.

Please wait to be told you may begin before turning this page.

This question paper consists of 21 printed pages and 3 blank pages.

PV2

✓ Non-ideal Solution vs Model Solution

✓ 3 step Analysis

✓ Reaction Circuits & Command term Constructed.

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✓ Related term.

* Hold yourself back even if you see a path.

→ Be lazy. If a path looks too algebraically heavy, it maybe is.

1 Given that $y = \dfrac{(1-3x)^2}{2x^{\frac{3}{2}}}$, which one of the following is a correct expression for $\dfrac{dy}{dx}$?

A $\frac{9}{4}x^{-\frac{1}{2}} + \frac{3}{2}x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{5}{2}}$

B $\frac{9}{4}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}} + \frac{3}{4}x^{-\frac{5}{2}}$

C $\frac{9}{4}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{5}{2}}$

D $-\frac{9}{4}x^{-\frac{1}{2}} + \frac{3}{2}x^{-\frac{3}{2}} + \frac{3}{4}x^{-\frac{5}{2}}$

E $-\frac{9}{4}x^{-\frac{1}{2}} + \frac{3}{2}x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{5}{2}}$

F $-\frac{9}{4}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{5}{2}}$

[X] Nominal solution: "in form of $\frac{P(x)}{Q(x)}$!"

formulaic thinking.

↓

"Quotient Rule."

→ Computational Hell...

Sol 9.) : yes it is $\frac{P(x)}{Q(x)}$ format, but P(x) and Q(x) are both polynomials, let me just rewrite it as.. $(2x^{\frac{3}{2}})^{-1} \times (1-3x)^2$

→ this is the critical thought here.

$= \frac{1}{2}x^{-\frac{3}{2}}(1-6x+9x^2)$

$= \frac{1}{2}x^{-\frac{3}{2}} + 3x^{-\frac{1}{2}} + 4.5x^{0.5}$.

→ Nxt Pg for TMUA 9.

2 $PQRS$ is a rectangle.

The coordinates of P and Q are $(0, 6)$ and $(1, 8)$ respectively.

The perpendicular to PQ at Q meets the x-axis at R.

What is the area of $PQRS$?

$\square PQRS = \sqrt{5} \times 8\sqrt{5} = 40$

A $\frac{5}{2}$

B $4\sqrt{10}$

C 20

D $8\sqrt{10}$

E 40

* You probably solved it as such.

$(\because y = f'(t)(x-t) + f(t))$

$\overline{PQ}: y = \frac{8-6}{1-0}(x-1) + f(1)$

$= 2(x-1) + 8$

$\overline{QR}: y = -\frac{1}{2}(x-1) + 8$

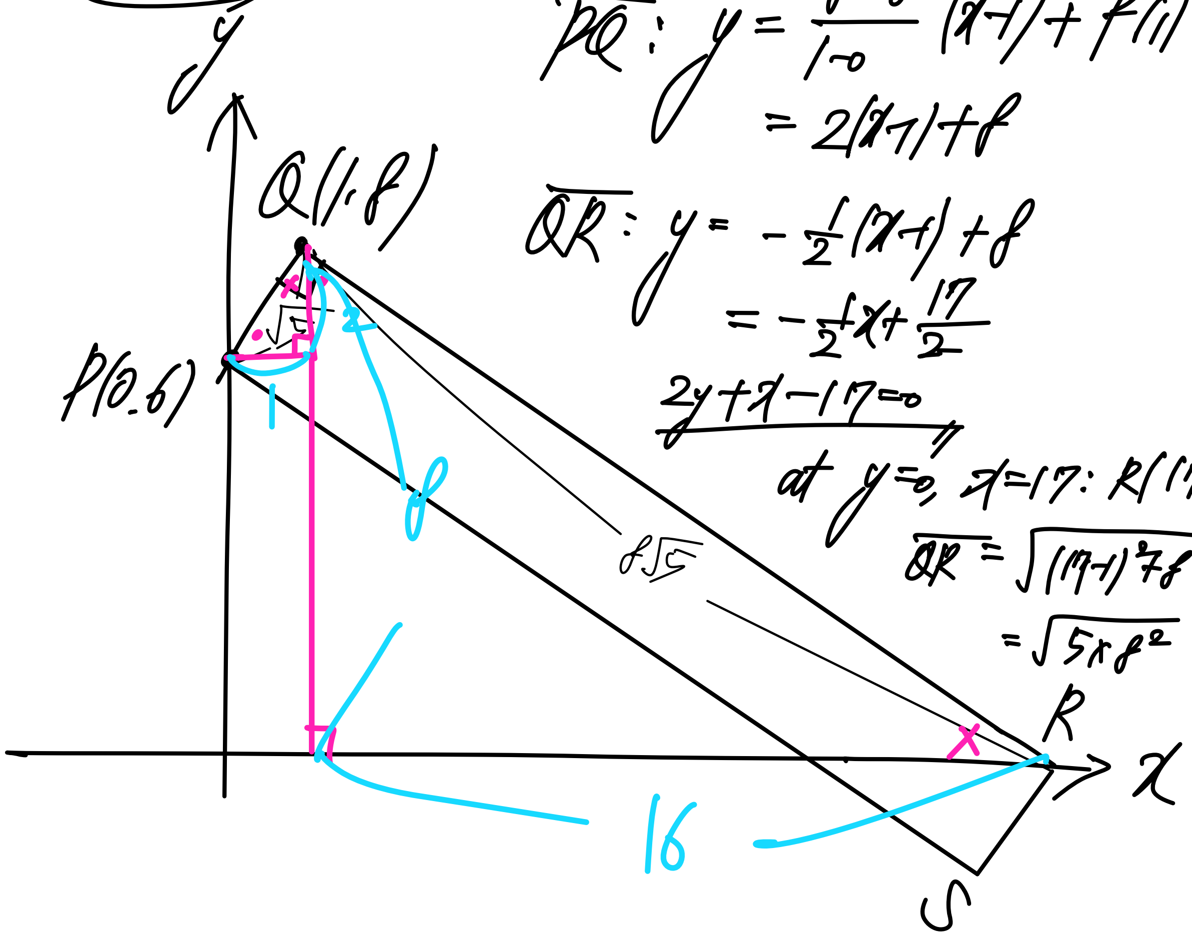
$= -\frac{1}{2}x + \frac{17}{2}$

$2y + x - 17 = 0$

at $y=0$, $x=17$: $R(17, 0)$

$\overline{QR} = \sqrt{(17-1)^2 + 8^2}$

$= \sqrt{5 \times 8^2} = 8\sqrt{5}$



* Kill the obsession to always sketch graph in relation to x&y axis.
→ There was no reason to specify the relationship between □PQRS and the axis here.

2 $PQRS$ is a rectangle.

The coordinates of P and Q are $(0, 6)$ and $(1, 8)$ respectively.

The perpendicular to PQ at Q meets the x-axis at R.

What is the area of $PQRS$?

A $\frac{5}{2}$

B $4\sqrt{10}$

C 20

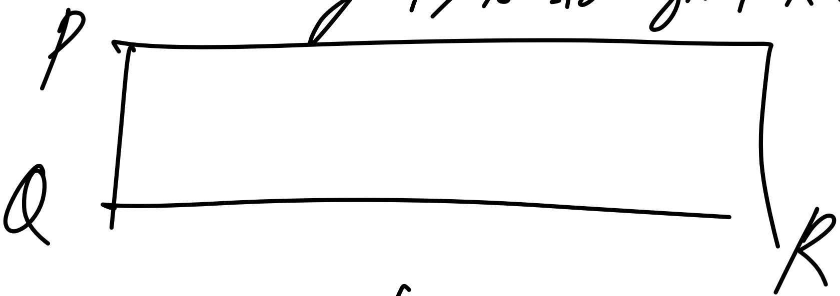
D $8\sqrt{10}$

E 40

Sol 9.) $\square PQRS = \overline{PQ} \times \overline{QR}$

$= \sqrt{5} \times \overline{QR}$

We know Q coordinate, so the goal, is to find R coordinate.



$QR: \dfrac{-1}{\text{Gradient of } \overline{PQ}}(x-1)+8$

$= \dfrac{-1}{\frac{8-6}{1-0}}(x-1)+8 = -\frac{1}{2}(x-1)+8$

at $y=0$, $\frac{1}{2}(x-1)=8$, so $R(17, 0)$

3 The first term of a geometric progression is $2\sqrt{3}$ and the fourth term is $\frac{9}{4}$

What is the sum to infinity of this geometric progression?

A $-2(2-\sqrt{3})$

B $4(2\sqrt{3}-3)$

C $\dfrac{16(8\sqrt{3}+9)}{37}$

D $\dfrac{4(2\sqrt{3}-3)}{7}$

E $\dfrac{4(2\sqrt{3}+3)}{7}$

F $2(2+\sqrt{3})$

G $4(2\sqrt{3}+3)$

$a_1 = 2\sqrt{3}$

$a_4 = \frac{9}{4}$

$\boxed{\begin{aligned} a_1 &= a_1 \\ a_4 &= a_1 r^3 \end{aligned}}$

$\boxed{\dfrac{a_4}{a_1} = r^3}$

$r = \dfrac{\sqrt{3}}{2}$

$\rightarrow \dfrac{a_4}{a_1} = r^3 = \dfrac{\left(\frac{9}{4}\right)}{2\sqrt{3}}$

this is what I mean when Don't write out long expressions & habituate simplifying it.

technically you can cross this step but to really cut exam time. It takes way too long.

$= \frac{9}{8} \times \frac{1}{\sqrt{3}}$

$= \dfrac{3\sqrt{3}}{8}$

$= \dfrac{(\sqrt{3})^3}{2^3}$

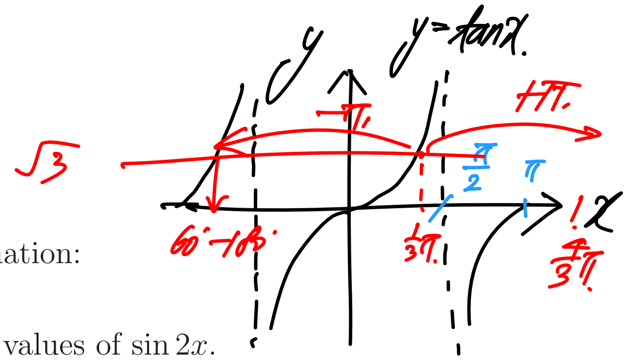
$= \left(\dfrac{\sqrt{3}}{2}\right)^3$

$a_n = ar^{n-1} = (2\sqrt{3}) \times \left(\dfrac{\sqrt{3}}{2}\right)^{n-1}$

$S_n = \dfrac{a}{1-r} = \dfrac{2\sqrt{3}}{1-\frac{\sqrt{3}}{2}} = \dfrac{4\sqrt{3}}{2-\sqrt{3}}$

$= 4\sqrt{3}(2+\sqrt{3})$

$= 4(2\sqrt{3}+3)$



4 The following question appeared in an examination:

$x = 60° + 180° \times n$

Given that $\tan x = \sqrt{3}$, find the possible values of $\sin 2x$.

$2x = 120° + 360° n$

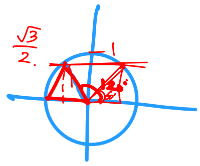
A student gave the following answer:

$\tan x = \sqrt{3}$ so $x = 60°$ and $2x = 120°$,

therefore $\sin 2x = \dfrac{\sqrt{3}}{2}$.

$\sin(2x) = \dfrac{\sqrt{3}}{2}$

Which one of the following statements is correct?



A $\dfrac{\sqrt{3}}{2}$ is the only possible value, and this is fully supported by the reasoning given in the student's answer.

B $\dfrac{\sqrt{3}}{2}$ is the only possible value, but the reasoning given should consider other possible values of x for which $\tan x = \sqrt{3}$.

C $\dfrac{\sqrt{3}}{2}$ is the only possible value, but the reasoning given should consider other possible values of x for which $\sin 2x = \dfrac{\sqrt{3}}{2}$.

D $\dfrac{\sqrt{3}}{2}$ is **not** the only possible value because the reasoning given should have considered other possible values of x for which $\tan x = \sqrt{3}$.

E $\dfrac{\sqrt{3}}{2}$ is **not** the only possible value because the reasoning given should have considered other possible values of x for which $\sin 2x = \dfrac{\sqrt{3}}{2}$.

$480 = 360 + 120$

$\sin(480°) = \sin(120°) = \sin(60°) = \dfrac{\sqrt{3}}{2}$

☆ The Blueprint for such logic questions:
Think in Venn Diagrams & Set Theory.

5 Consider the following three statements:

1 $10p^2 + 1$ and $10p^2 - 1$ are both prime when p is an odd prime. $p = 3$

2 Every prime greater than 5 is of the form $6n + 1$ for some integer n.

3 No multiple of 7 greater than 7 is prime.

The result $91 = 7 \times 13$ can be used to provide a counterexample to which of the above statements?

A none of them

B **1** only

C **2** only

D **3** only

E **1** and **2** only

F **1** and **3** only

G **2** and **3** only

H **1**, **2** and **3**

☆ logic of counterexample against if p then q.

→ If P, then NOT Q ?

2 things.

① Assume the condition to be true. (P)

② find A CASE s.t the statement is not true. (Q)

☆ "If its a sequence, Jot it out." #Command term: Sequence.

#Reaction Circuit:

6 A sequence $u_0, u_1, u_2, \ldots$ is defined as follows:

☆ It's a sequence!

Okay then I should probably

Just jot them out. ☆

Just jot out everything. That's the best way to start out sequences.

$$u_0 = 1$$

$$u_n = \int_0^1 4xu_{n-1}\,dx \quad \text{for } n \geqslant 1$$

What is the value of u_{1000}?

$$u_n = 2u_{n-1}\int_0^1 2x\,dx$$

$$= 2u_{n-1} \cdot [x^2]_0^1$$

$$= 2u_{n-1}$$

$$u_n = 2 \times u_{n-1} \quad n-1$$

$$u_n = a \times r^{n-1} = 2 \times 2^{n-1} = 2^n$$

$$\begin{cases} a = u_1 = 2 \\ r = 2. \end{cases}$$

A 2^{1000}

B 4^{1000}

C $\dfrac{4}{1000!}$

D $\dfrac{4}{1001!}$

E $\dfrac{2^{1000}}{1000!}$

F $\dfrac{4^{1000}}{1000!}$

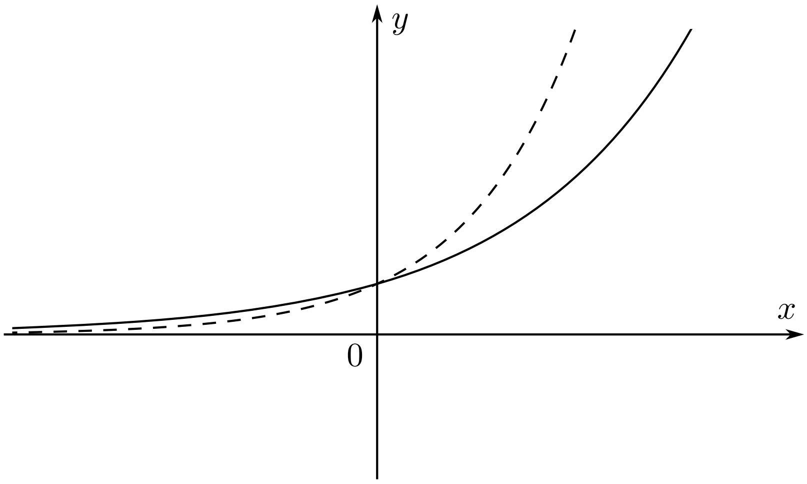
G $\dfrac{2^{1000}}{1001!}$

H $\dfrac{4^{1000}}{1001!}$

Inverse function. Logarithmic & Exponential...

7 The graphs of two functions are shown here:

- $y = a^x$ is shown with a solid line, where a is a positive real number
- $y = f(x)$ is shown with a dashed line



Which of the following statements (**1**, **2**, **3**, **4**) could be true?

1 $f(x) = b^x$ for some $b > a$
2 $f(x) = b^x$ for some $b < a$
3 $f(x) = a^{kx}$ for some $k > 1$
4 $f(x) = a^{kx}$ for some $k < 1$

A **1** only

B **2** only

C **3** only

D **4** only

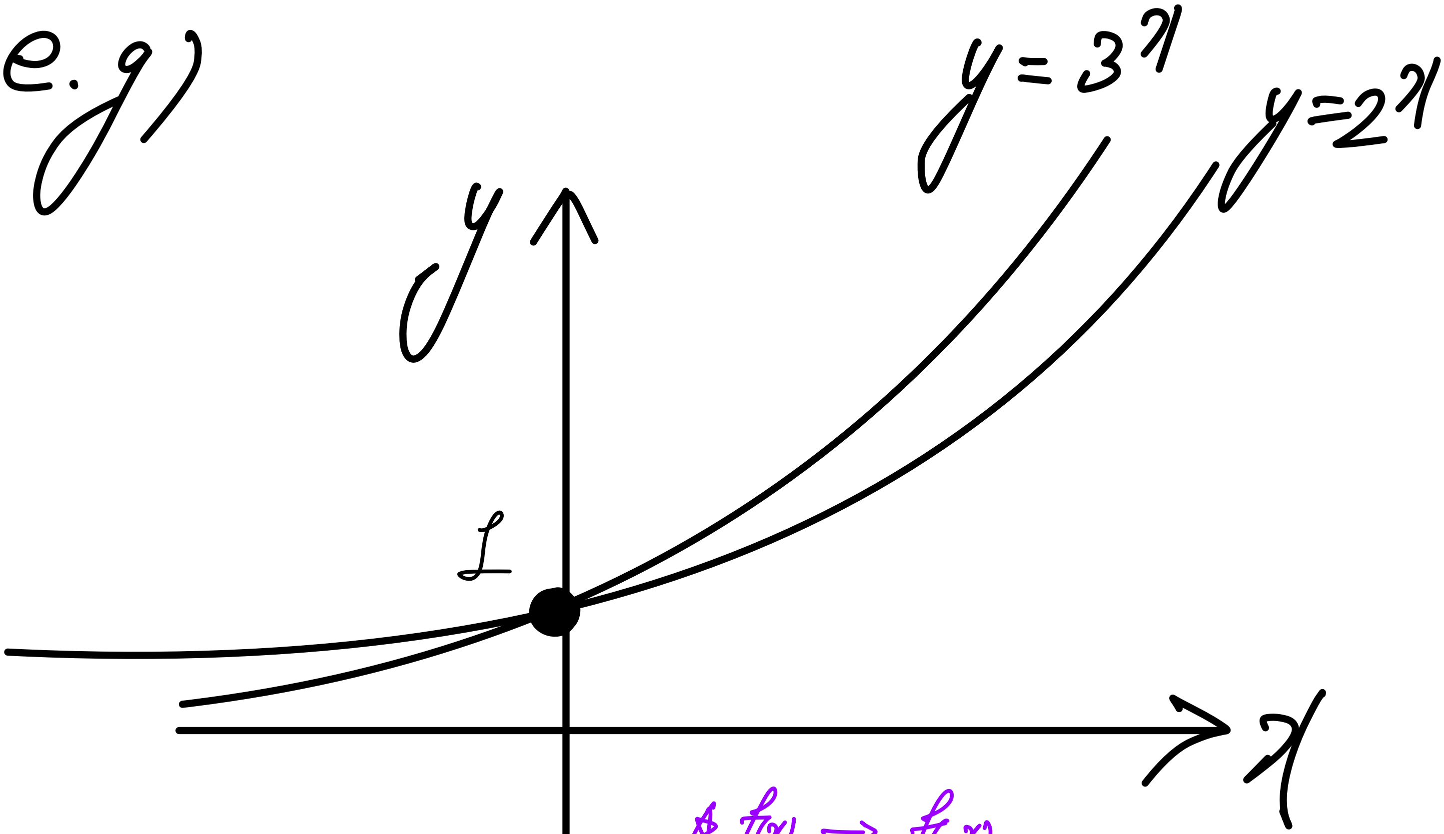
E **1** and **3** only

F **1** and **4** only

G **2** and **3** only

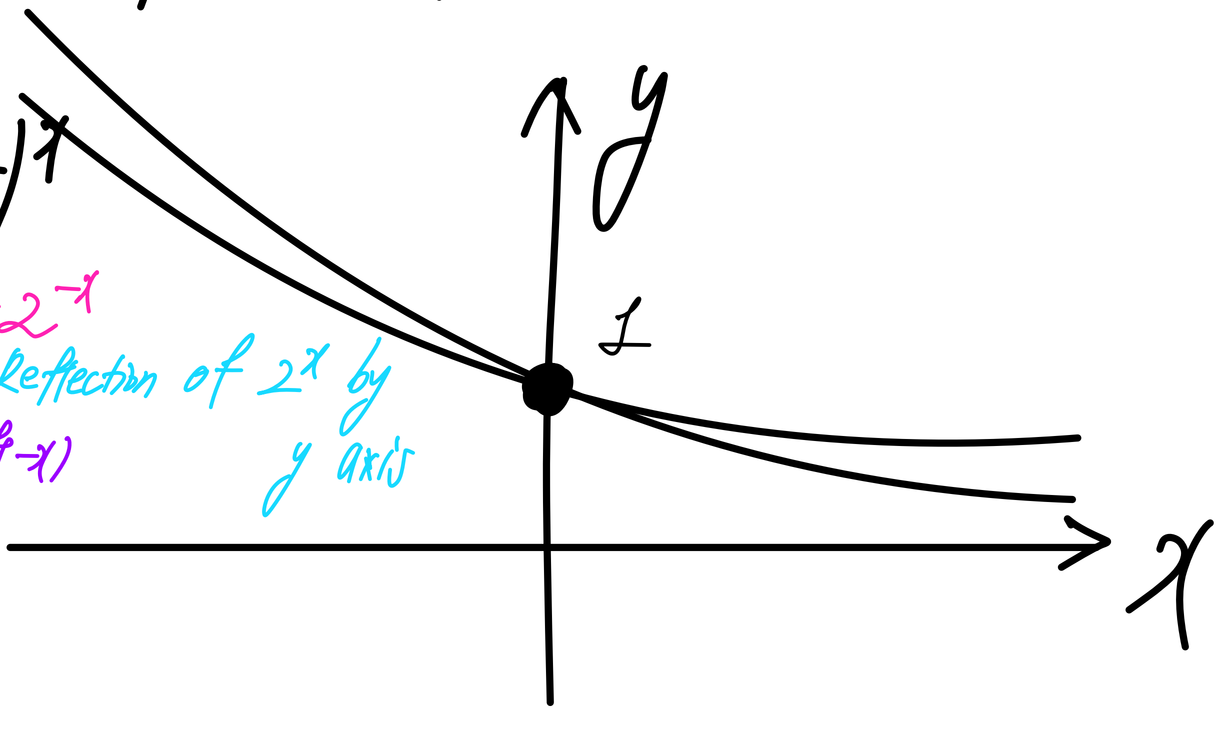
H **2** and **4** only

e.g)



☆ $f(x) \to f(-x)$

$y = \left(\frac{1}{3}\right)^x = 3^{-x}$ = Reflection of 3^x by y-axis.



$y = \left(\frac{1}{2}\right)^x$

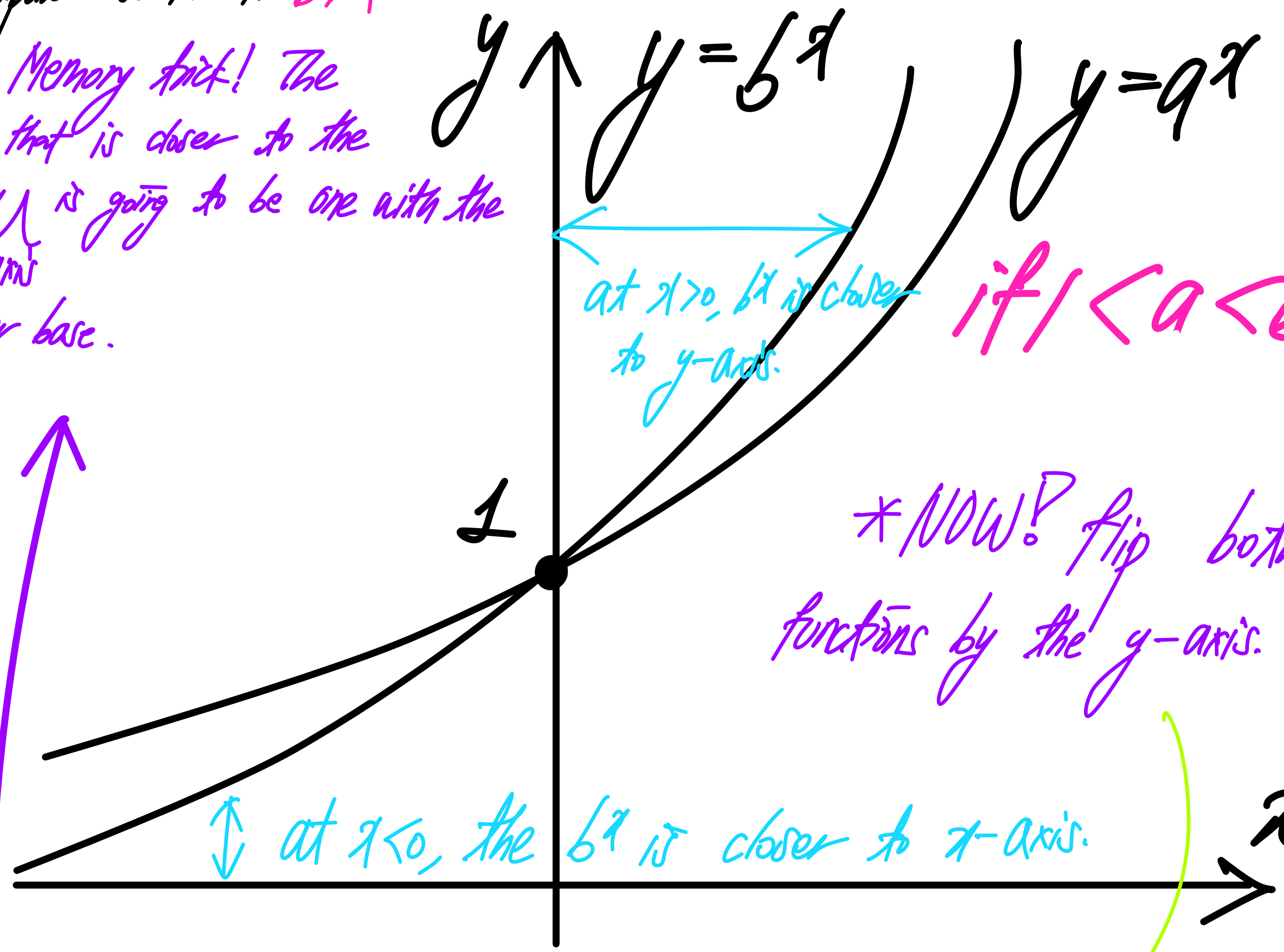
$= 2^{-x}$

= Reflection of 2^x by y axis

☆ $f(x) \to f(-x)$

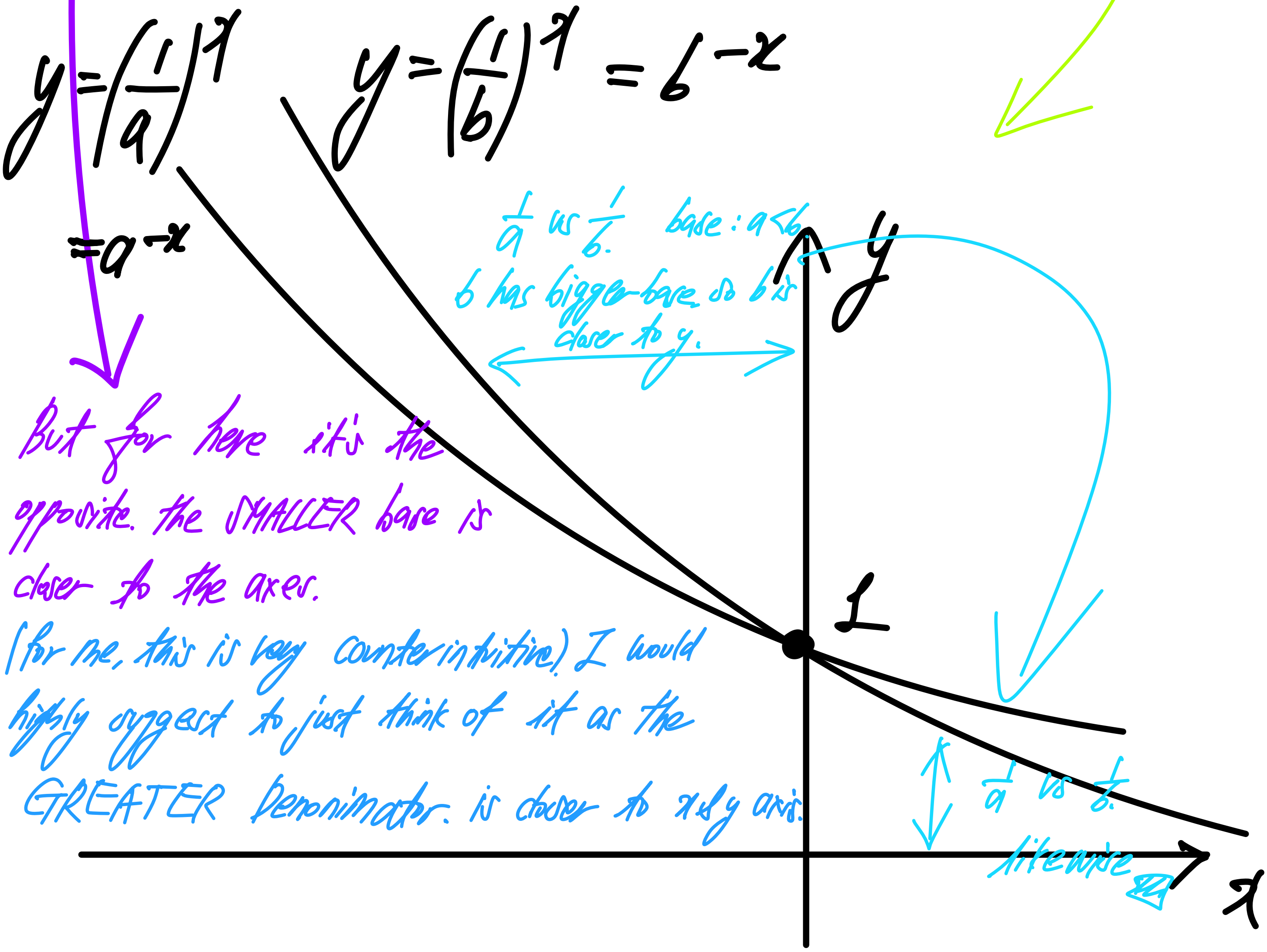
☆ Compare base for T/. $b>a$

→ Memory trick! The one that is closer to the x & y axis is going to be one with the bigger base.



if $1<a<b$.

* NOW! flip both functions by the y-axis.



But for here it's the opposite. the SMALLER base is closer to the axes.

(for me, this is very counterintuitive) I would highly suggest to just think of it as the GREATER Denominator is closer to x & y axis.

9. ☆ When comparing magnitudes by two methods.

Almost every one of them are ($A-B$ vs 0) or ($\frac{A}{B}$ vs 1).

8 Which one of the following numbers is smallest in value?

① Subtraction, or ② Division.

A $\log_2 7$

B $(2^{-3} + 2^{-2})^{-1}$

① $A > B \rightarrow A-B > B-B$

so $A-B$ vs 0.

C $2^{(\pi/3)}$

D $\dfrac{1}{4(\sqrt{2}-1)^3}$

② $A > B \rightarrow \frac{A}{B} > \frac{B}{B}$

$\frac{A}{B} > 1$

* Caution!
* Sign matters for inequalities

E $4\sin^2\left(\frac{\pi}{4}\right)$

9 Consider the following attempt to prove this true theorem:

Theorem: $a^3 + b^3 = c^3$ has no solutions with a, b and c positive integers.

Attempted proof:
Suppose that there are positive integers a, b and c such that $a^3 + b^3 = c^3$.

I We have $a^3 = c^3 - b^3$.

the number (so numerical value) of $c-b$, c^2+bc+b^2 itself can be factorised.

II Hence $a^3 = (c-b)(c^2+cb+b^2)$. 2x.

III It follows that $a = c - b$ and $a^2 = c^2 + cb + b^2$, since $a \leqslant a^2$ and $c - b \leqslant c^2 + cb + b^2$.

Realise that. factorisation ≠ prime factorisation.

e.g) $12 = 3\times4 = 2\times6 = 1\times12$.

IV Eliminating a, we have $(c-b)^2 = c^2 + cb + b^2$.

V Multiplying out, we have $c^2 - 2cb + b^2 = c^2 + cb + b^2$.

VI Hence $3cb = 0$ so one of b and c is zero.

But this is a contradiction to the original assumption that all of a, b and c are positive. It follows that the equation has no solutions.

We Don't know what it which.

Comment on this proof by choosing one of the following options:

A The proof is correct

B The proof is incorrect and the first mistake occurs on line I.

C The proof is incorrect and the first mistake occurs on line II.

D The proof is incorrect and the first mistake occurs on line III.

E The proof is incorrect and the first mistake occurs on line IV.

F The proof is incorrect and the first mistake occurs on line V.

G The proof is incorrect and the first mistake occurs on line VI.

★ If you don't have Good Algorithm for solving Necessary & sufficient Algorithm, it will continue to confuse you.

→ The proper way to think of necessary / sufficient is also Venn Diagrams & Set theory.

① $A \to B$

$B \to A$

10 ★ $f(x)$ is a function defined for all real values of x.

Which one of the following is a **sufficient** condition for $\int_1^3 f(x)\,dx = 0$?

A B

so * $A \to B$ ... which means $A \subset B$.

(A 가 총 쏘니까 총쏜조건 thus sufficient).

A $f(2) = 0$

B $f(1) = f(3) = 0$

C $f(-x) = -f(x)$ for all x

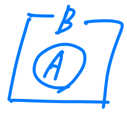
D $f(x+2) = -f(2-x)$ for all x

E $f(x-2) = -f(2-x)$ for all x

< Algorithm for necessary / sufficient condition >

[1] Identify what set is subset of what.

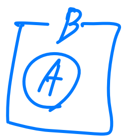
A is sufficient for B: $A \to B$ which means $A \subset B$.



난 이렇게 외움. A가 B한테 총을 쏘니까

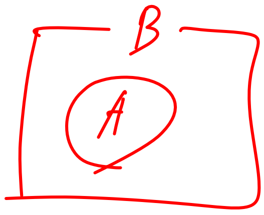
A가 총쏜조건 (which kinda sounds like 충분조건 = sufficient condition)

B is necessary for A: $A \to B$ which means $A \subset B$. same thing.



B가 총맞고 피 흘려서 필요조건. so necessary condition ... lol.

[2]



$A \to B$ so if A is true B is true.

Also, just because B is true doesn't mean A is true.

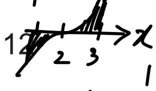
$$\left.\begin{aligned} f(x+k) + f(2a-k-x) &= 2b \\ f(x) + f(2a-x) &= 2b \\ \text{cf) D. E} \quad f(a+x) + f(a-x) &= 2b. \end{aligned}\right\}$$

* all 3 mean the same thing.

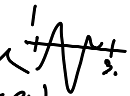
$f(x)$ is point symmetry about (a, b)

in D. $f(2+x) + f(2-x) = 0$ so $f(x)$ is $(2, 0)$ point symmetric.

if D is true, then yeah obviously $\int_1^3 f(x)\,dx = 0$.



But just because $\int_1^3 f(x)\,dx = 0$ Doesn't mean $f(x)$ is point symmetric.



Counter ex).

9. if $f(☆) + f(\triangle) = 2b$ "and" $☆ + \triangle = 2a$, (CONSTANT.) then $f(x)$ is point symmetric about point $P(a, b)$

10 $f(x)$ is a function defined for all real values of x.

Which one of the following is a **sufficient** condition for $\int_1^3 f(x)\,dx = 0$?

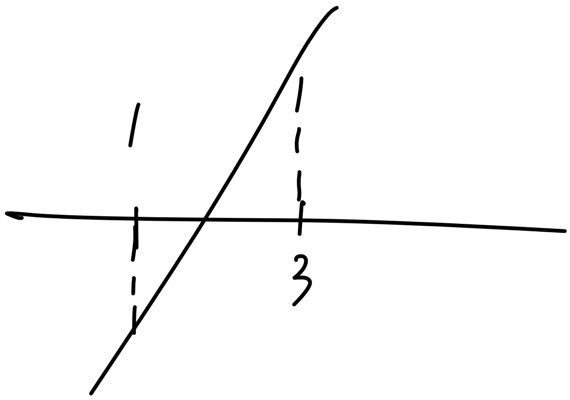
A $f(2) = 0$

B $f(1) = f(3) = 0$

C $f(-x) = -f(x)$ for all x

D $f(x + 2) = -f(2 - x)$ for all x

E $f(x - 2) = -f(2 - x)$ for all x



D ☆ When $f(☆) + f(\triangle) = 2b$, and $☆ + \triangle = 2a$, then $f(x)$ is point symmetric about (a, b)

$\underbrace{f(x+2)}_{☆} + \underbrace{f(2-x)}_{\triangle} = 0$ $2b = 0$

$\frac{☆ + \triangle}{2} = \frac{2 - x + 2 + x}{2} = 2 = a$ $\rightarrow (a.b) = (2.0)$

E $\underbrace{f(x-2)}_{☆} + \underbrace{f(2-x)}_{\triangle} = 0.$

$☆ + \triangle =$ Constant of zero!

$\therefore$ this is point symmetric about the origin $\otimes$

* The key was to translate it & Compartmentalise entire area into two areas seperately.

11 The function $f(x)$ is increasing and $f(0) = 0$.

The positive constants a and b are such that $a < b$.

The area of the region enclosed by the curve $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ is denoted by R.

The function $g(x)$ is defined by $g(x) = f(x) + 2f(b)$.

Which of the following is an expression for the area enclosed by the curve $y = g(x)$, the x-axis and the lines $x = a$ and $x = b$?

A $R + (b - a)f(b)$

B $R + 2(b - a)f(b)$ → $\underline{\underline{R}} + \underbrace{2f(b)}_{\text{\# height of rectangle}} \times \underbrace{(b-a)}_{\text{\# base of rectangle.}}$.

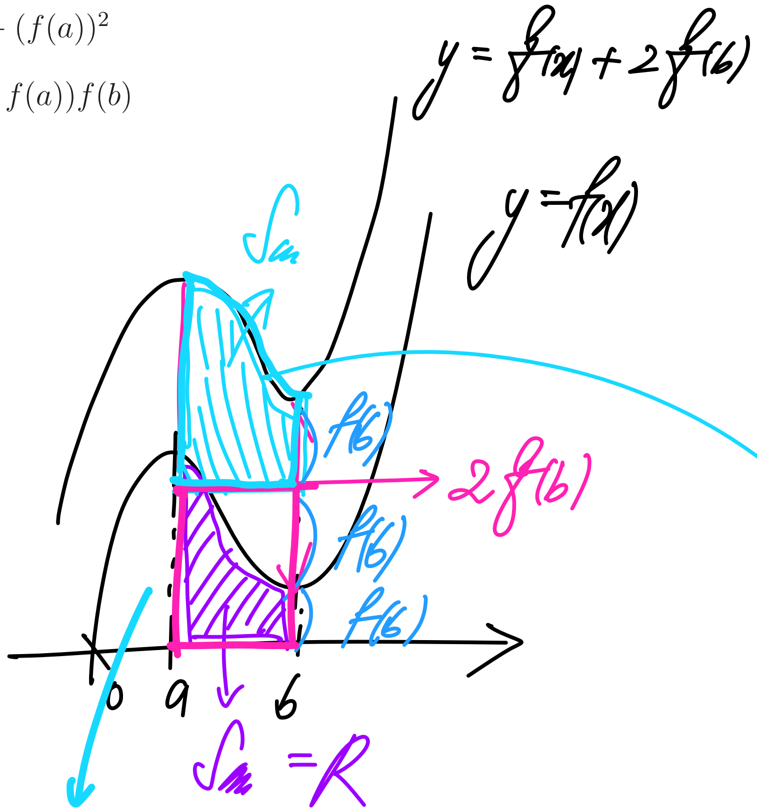
C $R + 2f(b) - f(a)$

D $R + 2f(b)$

E $R + (f(b))^2$

F $R + (f(b))^2 - (f(a))^2$

G $R + 2(f(b) - f(a))f(b)$

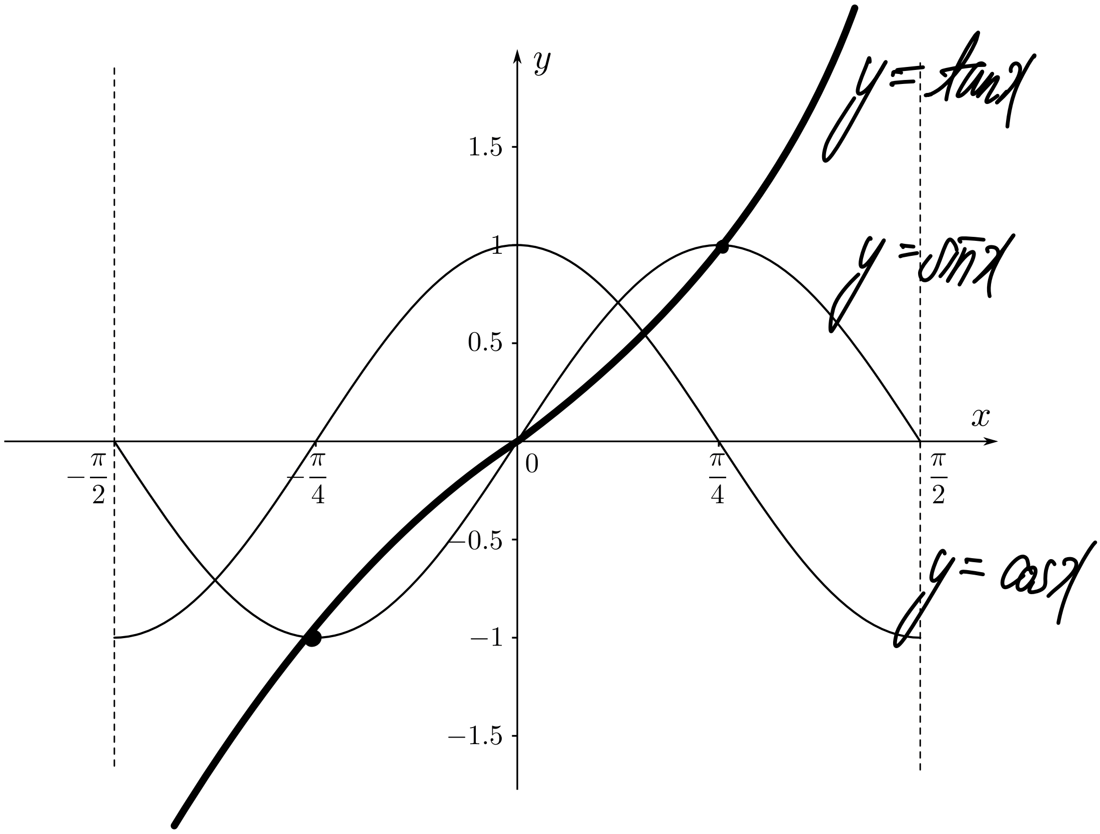


Total Sum = (Pink Area of Rectangle) + Area of S_m

* $y = \sin kx$
$\tan kx$
* $y = \cos kx$

these show up A LOT. As they are $f(x) \to f(\frac{1}{k}x)$ s.t $f(x)$ = trig functions.
(scale factor of k)
→ Habituate visualisation of stretching & squishing gummy worms!

12 The diagram shows the graphs of $y = \sin 2x$ and $y = \cos 2x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$



Which one of the following is **not** true? Just try. Sub each in. (Brute force method.)

A $\cos 2x < \sin 2x < \tan x$ for some real number x with $-\frac{\pi}{2} < x < \frac{\pi}{2}$

B $\cos 2x < \tan x < \sin 2x$ for some real number x with $-\frac{\pi}{2} < x < \frac{\pi}{2}$

C $\sin 2x < \cos 2x < \tan x$ for some real number x with $-\frac{\pi}{2} < x < \frac{\pi}{2}$

D $\sin 2x < \tan x < \cos 2x$ for some real number x with $-\frac{\pi}{2} < x < \frac{\pi}{2}$

E $\tan x < \sin 2x < \cos 2x$ for some real number x with $-\frac{\pi}{2} < x < \frac{\pi}{2}$

F $\tan x < \cos 2x < \sin 2x$ for some real number x with $-\frac{\pi}{2} < x < \frac{\pi}{2}$

I call this VA. Verbally Articulation.

// Unless... you read out a pattern (this is what I did). So you "read" out what you see:
The MCQ. are all comparing magnitudes of 3 functions: $\cos 2x$, $\sin 2x$, $\tan x$!
So I could sketch each and compare.

☆ implies a, b. and c are greater than 1.

13 The positive real numbers $a \times 10^{-3}$, $b \times 10^{-2}$ and $c \times 10^{-1}$ are each in standard form, and

$$(a \times 10^{-3}) + (b \times 10^{-2}) = (c \times 10^{-1}).$$

Standard form means that $1 \leq A < 10$

☆ key command term:

$*$ $A \times 10^n$ is in standard form when $1 \leq A < 10$.

Which of the following statements (I, II, III, IV) **must** be true?

$a + 10b = 100c$

~~I $a > 9$~~

II $b > 9$

~~III $a < c$~~

~~IV $b < c$~~

So $\begin{pmatrix} a \times 10^{☆} \\ b \times 10^{☆} \\ c \times 10^{☆} \end{pmatrix}$ form simply implies $a, b, c \in [1, 10)$

A I only

B II only

C I and II only

D I and III only

E I and IV only

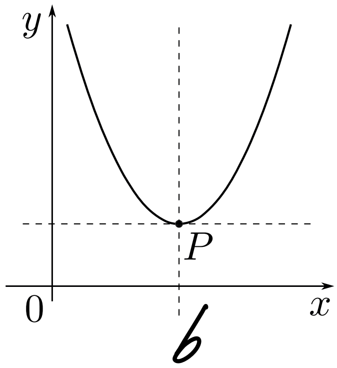
F II and III only

G II and IV only

H I, II, III and IV

14 The diagram below shows the graph of $y = x^2 - 2bx + c$. The vertex of this graph is at the point P.

$(x-b)^2 + c - b^2.$

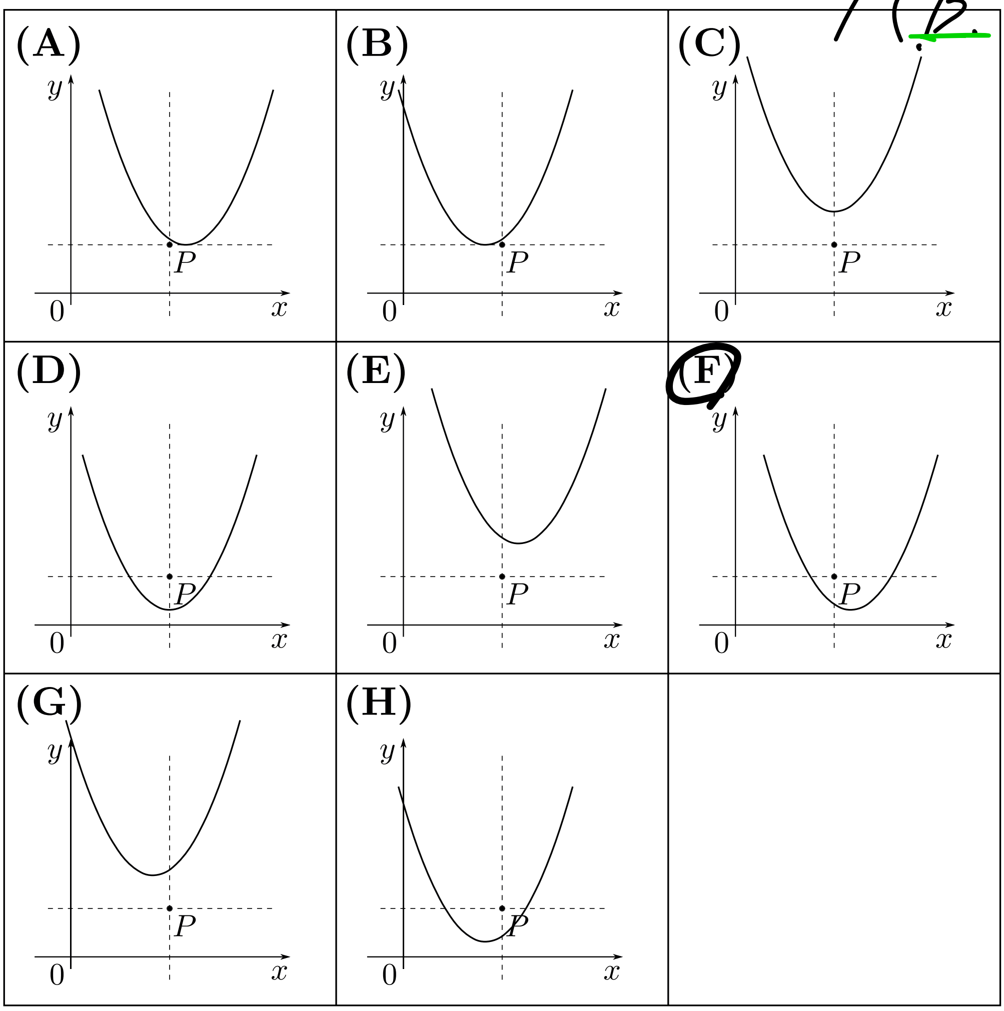


$P(b, c-b^2).$

$c - b^2.$

Which one of the following could be the graph of $y = x^2 - 2Bx + c$, where $B > b$?

$P'(B, c-B^2)$



Just try out writing for sequences

Sequences

you may ask: Why did I choose to view f(x) as a sequence. It's because A sequence is a subset of graphs. (↗ 2017 Paper 1 Sequences)

15 The function f is defined on the positive integers as follows:

$$f(1) = 5, \text{ and for } n \geqslant 1: \qquad \begin{array}{ll} f(n+1) = 3f(n) + 1 & \text{if } f(n) \text{ is odd} \\ f(n+1) = \frac{1}{2}f(n) & \text{if } f(n) \text{ is even} \end{array}$$

The function g is defined on the positive integers as follows:

$$g(1) = 3, \text{ and for } n \geqslant 1: \qquad \begin{array}{ll} g(n+1) = g(n) + 5 & \text{if } g(n) \text{ is odd} \\ g(n+1) = \frac{1}{2}g(n) & \text{if } g(n) \text{ is even} \end{array}$$

What is the value of $f(1000) - g(1000)$?

A -6

B -5

C 1

D 2

E 4

F 8

The thought process: WTF → it has n(integer) as domain! It must be a sequence. → My Reaction Circuit of sequences are to jot them out 😵

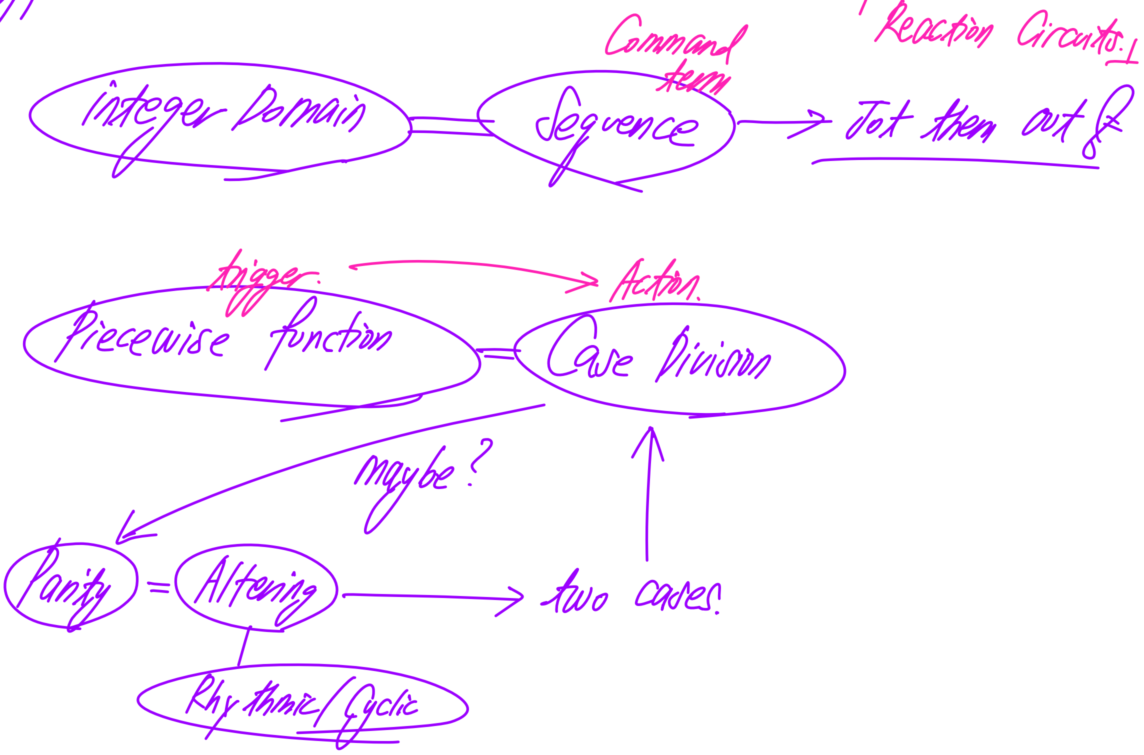
$f(x)$: $5 \to 16 \to 8 \to 4 \to 2 \to 1 \to 4 \to 2 \to 1$

$g(x)$: $3 \to 5 \to 8 \to 4 \to 2 \to 1 \to 4 \to 2 \to 1 \ \cdots$

9. there are a lot of questions such that they give a random algorithm (it could be an inductive question like ↗ TMUA9. 2017 Paper 1 #11) or a piecewise function like this one). → for these type of functions, the answers tend to have the pattern of: first couple of terms in the sequence being sporadic (flying all over the place) but eventually become cyclical.

* It does help to visualise it into some neural networks & make the connections. (No need for the mindmap to fit perfectly).

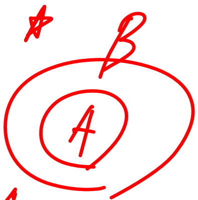
e.g)



Try it out! Find how each concept all coalescene into unity!

* If A then B

* $A \rightarrow B$.   * $A \subset B$



A is smaller B is larger

16 Consider the following statement:

again all the same stuff.

(∗) **If** $f(x)$ is an integer for every integer x, **then** $f'(x)$ is an integer for every integer x.

Which one of the following is a **counterexample** to (∗)?

A $f(x) = \dfrac{x^3 + x + 1}{4}$

B $f(x) = \dfrac{x^4 + x^2 + x}{2}$

C $f(x) = \dfrac{x^4 + x^3 + x^2 + x}{2}$

D $f(x) = \dfrac{x^4 + 2x^3 + x^2}{4}$

Only $\left\{ \begin{array}{l} \underline{A \not\rightarrow B} \\ \quad = \triangle \\ \underline{B' \rightarrow A} \text{ (Contrapositive)} \\ \quad = ☆ \end{array} \right\}$ are valid candidates for counterexamples.

If and only if AKA "iff". It means A = B.
A → B and B → A. Which means {A }

17 A set S of whole numbers is called *stapled* **if and only if** for every whole number a which is in S there exists a prime factor of a which divides at least one other number in S.

Let T be a set of whole numbers. Which of the following is true **if and only if** T is **not** stapled?

A For every number a which is in T, there is no prime factor of a which divides every other number in T.

B For every number a which is in T, there is no prime factor of a which divides at least one other number in T.

C For every number a which is in T, there is a prime factor of a which does not divide any other number in T.

D For every number a which is in T, there is a prime factor of a which does not divide at least one other number in T.

E There exists a number a which is in T such that there is no prime factor of a which divides every other number in T.

F There exists a number a which is in T such that there is no prime factor of a which divides at least one other number in T.

G There exists a number a which is in T such that there is a prime factor of a which does not divide any other number in T.

H There exists a number a which is in T such that there is a prime factor of a which does not divide at least one other number in T.

☆ Negating statements.

★ The mathematical definition of increasing / Decreasing.

if $x_1 < x_2$, then $f(x_1) < f(x_2)$ if and only if $f(x)$ is increasing at $[a,b]$,

if $x_1 < x_2$, then $f(x_1) > f(x_2)$ if and only if $f(x)$ is decreasing at $x_1, x_2 \in [a,b]$ (likewise)

18 Consider the following problem:

Solve the inequality $\left(\frac{1}{4}\right)^n < \left(\frac{1}{32}\right)^{10}$, where n is a positive integer.

A student produces the following argument:

$$\left(\frac{1}{4}\right)^n < \left(\frac{1}{32}\right)^{10}$$

$\log_{\frac{1}{2}} x = -\log_2 x$

x

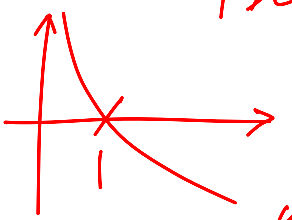
(I)

$$\log_{\frac{1}{2}}\left(\frac{1}{4}\right)^n < \log_{\frac{1}{2}}\left(\frac{1}{32}\right)^{10}$$

(Decreasing function)

(II)

$$n\log_{\frac{1}{2}}\left(\frac{1}{4}\right) < 10\log_{\frac{1}{2}}\left(\frac{1}{32}\right)$$



$y = -\log_2 x$

(III)

$$n < \frac{10\log_{\frac{1}{2}}\left(\frac{1}{32}\right)}{\log_{\frac{1}{2}}\left(\frac{1}{4}\right)}$$

(IV)

$$n < \frac{10 \times 5}{2} = 25$$

(V)

$$1 \leqslant n \leqslant 24$$

Which step (if any) in the argument is invalid?

A There are no invalid steps; the argument is correct

B Only step (I) is invalid; the rest are correct

C Only step (II) is invalid; the rest are correct

D Only step (III) is invalid; the rest are correct

E Only step (IV) is invalid; the rest are correct

F Only step (V) is invalid; the rest are correct

"What is sufficient for B" ≠ "What makes B sufficient?"

↳ $A \to B$ $\qquad B \to A$

Sufficient for B, so $A \to B$. and finding A.

19 Which one of the following is a **sufficient** condition for the equation $\boxed{x^3 - 3x^2 + a = 0}$, where a is a constant, to have exactly one real root?

B

A $a > 0$

B $a \leqslant 0$

C $a \geqslant 4$

D $a < 4$

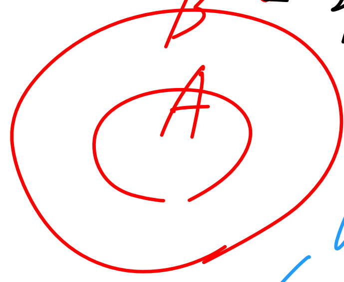
E $|a| > 4$

F $|a| \leqslant 4$

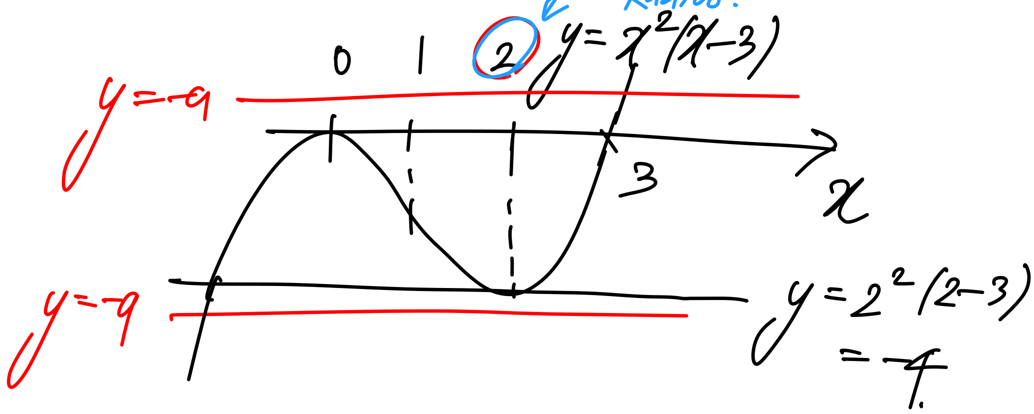
G $a = \frac{9}{4}$

H $|a| = \frac{3}{2}$

$B = x^3 - 3x^2 + a = 0.$



☐ Why is this 2?
We learned this from Ratios.



$y = -a > 0$ or $-a < -4.$

★ Pure brain teasers, lay them out rather than scribbling.
★. Write out potential scenarios.

20 I have forgotten my 5-character computer password, but I know that it consists of the letters a, b, c, d, e in some order. When I enter a potential password into the computer, it tells me exactly how many of the letters are in the correct position.

When I enter `abcde`, it tells me that none of the letters are in the correct position. The same happens when I enter `cdbea` and `eadbc`.

Using the best strategy, how many **further** attempts must I make in order to **guarantee** that I can **deduce** the correct password?

A None: I can deduce it immediately

B One

C Two

D Three

E More than three

sol me) a b c d e
c d b e a
e a d b c
↓ ↓ ↓ ↓ ↓
b c a a b
d e e c d
① ② ③ ④ ⑤

① . ⑤ = (b, d) A or (d, b) A'
②, ③. ④ = (c, e, a) B
(e, a. c) B'

So its binary thus try any set if it's valid.

A/A'
B/B'

END OF TEST

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