TMUA9.
MOCK ALPHA.

One set, Paper 1 and Paper 2, TMUA format.

Free. No NOIR PREMIERE required.
Official time 75 minutes. NOIR target: 15.

TMUA9.

How to use this mock

One set: a Paper 1 (Applications of Mathematical Knowledge) and a Paper 2 (Mathematical Reasoning), twenty multiple-choice questions each, in the TMUA format: pick one letter, no working marked, no calculator.

The point. The official paper gives you 75 minutes. A NOIR-trained student should be done in 15. Every question is one reaction circuit with nothing else in the way, named in cyan at the end of the question, except the last few in each paper, which chain two or three ideas together and are left unnamed on purpose: that is the part of the paper built to feel like a real Q15-20.

The rule. Set the timer for 15 minutes per paper. Stop when it rings, whatever is left is a miss. Mark from the key at the back: every answer comes with the circuit and the one-line reflex.

Verification. Every numeric answer was recomputed from the underlying integral, root, or coordinate geometry by a computer algebra system, independently of the shortcut; every logic answer was checked by brute force where a predicate could be evaluated.

Want more of this. This is one of eight free sets in the TMUA9 Question Bank. The full ten-set TMUA9. Papers, the DELUXE rehearsal, and NOIR PREMIERE go further: `https://tmua9.com/noir-premiere`.

Set 1

Paper 1 Applications of Mathematical Knowledge

TMUA9. PAPERS

20 questions. Official time 75 minutes. NOIR target: 15 minutes. Every question is one reaction circuit, named in the margin. Answer on the grid at the end of the set.

1 What is the sum of all solutions of the equation $\cos 2x = -\frac{5}{6}\cos x - \frac{2}{3}$ in the interval $0 \le x \le 2\pi$? [root-pair sum]

A π **B** 2π **C** 3π **D** 4π **E** $\frac{16\pi}{3}$ **F** 12π

2 A straight line meets the curve $y = 3x^3 - 4x^2 + cx + d$ at the points where $x = 3$, $x = -4$ and $x = k$. What is k? [root sum]

A $-\frac{17}{3}$ **B** -3 **C** $-\frac{1}{3}$
D $\frac{1}{3}$ **E** $\frac{7}{4}$ **F** $\frac{7}{3}$

3 How many distinct real roots does the equation $2x^3 - 12x^2 + 18x + (-11) = 0$ have? [root band]

A 0 **B** 1 **C** 2 **D** 3

4 What is the value of $\sum_{k=0}^{2025} \sin(10 + 90k)^\circ$? [seasons]

A $\cos 10^\circ$ **B** $\sin 10^\circ + \cos 10^\circ$
C $\sin 10^\circ$ **D** $\sin 10^\circ - \cos 10^\circ$
E $-\cos 10^\circ$ **F** $2\sin 10^\circ$

5 The function $f(x) = \cos(\frac{\pi x}{p})$ is considered only on the closed interval $[7, 15]$. For natural numbers p and q with $7 \le q \le 15$, f attains its maximum value 1 at $x = q$. What is the least possible value of $p + q$? [$p+q$ search]

A 7 **B** 8 **C** 9 **D** 10 **E** 11 **F** 15

6 The tangent to the curve $y = x^3 + bx$ (where b is a constant) at the point where $x = -3$ passes through the point $(1, 80)$, which does not lie on the curve. What is the x-intercept of this tangent? [ext. tangent]

A $-\frac{108}{13}$ **B** -3 **C** $-\frac{27}{13}$
D $-\frac{81}{52}$ **E** $-\frac{9}{13}$ **F** 1

7 What is $\int_5^6 (x^2 + 2x - 1)\,dx$? [group]

A $\frac{121}{4}$ **B** $\frac{118}{3}$ **C** $\frac{121}{3}$
D $\frac{124}{3}$ **E** 102 **F** 484

8 What is the sum of all solutions of the equation $\cos 2x = \frac{8}{3}\cos x - \frac{5}{3}$ in the interval $0 \le x \le 2\pi$? [root-pair sum]

A π **B** 2π **C** 3π **D** 4π **E** 6π **F** 16π

9 What is the smallest positive value of a for which the line $x = a$ is a line of symmetry of the curve $y = \sin(4x + \frac{\pi}{4})$? [trig sym]

A $\frac{\pi}{32}$ **B** $\frac{\pi}{16}$ **C** $\frac{\pi}{8}$
D $\frac{3\pi}{16}$ **E** $\frac{\pi}{4}$ **F** $\frac{5\pi}{16}$

10 The roots of $x^2 - 13x + c = 0$ differ by 5. What is c? [Vieta]

A -36 **B** 3 **C** 6 **D** 34 **E** 36 **F** 37

11 In triangle ABC, the cevians AD, BE and CF meet at a single point, with $\dfrac{BD}{DC} = \frac{3}{2}$ and $\dfrac{CE}{EA} = \frac{1}{2}$. What is $\dfrac{AF}{FB}$? [Ceva]

A $\frac{1}{3}$ **B** $\frac{1}{2}$ **C** $\frac{3}{4}$
D $\frac{4}{3}$ **E** $\frac{8}{3}$ **F** 3

12 The curve $y = 18x^3 + bx^2 + cx + d$ touches the line $y = k$ at $x = 999$ and crosses it at $x = 1004$. What is the area of the region enclosed between the curve and the line? [$\bigstar^4$/12]

A $\frac{625}{4}$ **B** $\frac{1875}{8}$ **C** 625
D $\frac{1873}{2}$ **E** $\frac{1875}{2}$ **F** 11250

13 The tangent to the curve $y = x^3 + bx$ (where b is a constant) at the point where $x = 2$ passes through the point $(-1, -20)$, which does not lie on the curve. What is the x-intercept of this tangent? [ext. tangent]

A -4 **B** -1 **C** 1 **D** 2 **E** 4 **F** 12

14 A cubic curve has turning points at $x = -1961$ and $x = -1951$, and its point of inflection is $(-1956, 7)$. What is the value of the integral of the cubic between $x = -1961$ and $x = -1951$? [five points]

A 0 **B** 35 **C** 70 **D** 71 **E** 140 **F** 280

15 Functions f and g are defined for all real x by $f(x) = 1x|x|$ and

$$g(x) = \begin{cases} a(x+2) & x < -2 \\ 0 & -2 \le x \le 2 \\ a(x-2) & x > 2 \end{cases}$$

where $a > 0$ is a constant. Let $h(x) = \int_0^x (g(u) - f(u))\, du$. The largest value of a for which h attains a local maximum or a local minimum value at exactly one value of x is k. When $a = k$, what is $k + h(4)$? [piecewise extremum]

A $-\frac{16}{3}$ **B** $\frac{8}{3}$ **C** 8 **D** 10 **E** $\frac{40}{3}$ **F** 16

16 Two circles, of radii 25 and 4, touch one another and both touch a common straight line ℓ, on the same side of ℓ. Circles $C_1, C_2, C_3, \ldots$ are then inscribed in the successive gaps: each C_n touches ℓ, touches the circle of radius 25, and touches C_{n-1} (with C_0 the circle of radius 4). For how many values of n does C_n have radius greater than $\frac{1}{16}$? [circle chain]

A 16 **B** 17 **C** 18 **D** 68 **E** 102 **F** 289

17 Triangle ABC has $BC = a$, $CA = b$, $AB = c = 13$, and $a \cos A = b \cos B$. Given also that $a + b = 17$, what is the largest possible area of the triangle? [two-branch cosine rule]

A $\frac{13\sqrt{30}}{24}$ **B** $\frac{13\sqrt{30}}{4}$
C 30 **D** $\frac{13\sqrt{30}}{2}$
E $\frac{60+13\sqrt{30}}{2}$ **F** $\frac{221}{2}$

18 Three distinct integers $a < b < c$ satisfy

$$a + b + c = 17, \qquad a^2 + b^2 + c^2 = 197.$$

What is the middle of the three? [mean-shift Diophantine]

A -2 **B** 7 **C** 8 **D** 12 **E** 17 **F** 84

19 $f(x) = x^2 - 2$. How many real solutions does $\underbrace{f(f(\cdots f}_{4}(x)\cdots)) = x$ have? [iterate + substitution]

A 8 **B** 15 **C** 16 **D** 17 **E** 24 **F** 48

20 For each integer $n \geq 1$ let $f(n)$ be the number of lists of *distinct* positive integers which begin with 1, end with n, and in which every term except the last divides the term after it. (For example $f(1) = 1$, $f(2) = 1$ and $f(6) = 3$, the three lists being $1, 6$ and $1, 2, 6$ and $1, 3, 6$.) How many integers n with $1 < n \leq 150$ satisfy $f(n) = 20$?

[divisor chain]

A 2 **B** 6 **C** 7 **D** 8 **E** 9 **F** 32

Set 1 Paper 2 Mathematical Reasoning

TMUA9. PAPERS

20 questions. Official time 75 minutes. NOIR target: 15 minutes. Every question is one reaction circuit, named in the margin. Answer on the grid at the end of the set.

1 Which of the following is the negation of the statement "There is an integer n such that $n^2 < 2n$."? [negate]

A For every integer n, $n^2 > 2n$.

B For every integer n, $n^2 \geq 2n$.

C There is no integer n such that $n^2 \geq 2n$.

D There is an integer n such that $n^2 \geq 2n$.

E There is an integer n such that $n^2 > 2n$.

F For every integer n, $n^2 < 2n$.

2 Claim: $\int_{-1}^{1} \frac{1}{x^2}\,\mathrm{d}x = -2$. Consider the following argument.

1 An antiderivative of $\frac{1}{x^2}$ is $-\frac{1}{x}$

2 so the integral equals $[-\frac{1}{x}]_{-1}^{1}$

3 $= -1 - (1) = -2$

4 hence the claim holds

Which of the following best describes the argument? [proof]

A The argument is incorrect: the first error is in line 2

B The argument is incorrect: the first error is in line 1

C The argument is incorrect: the first error is in line 3

D The argument is incorrect: the first error is in line 4

E The argument is correct

3 Which of the following statements is/are true?

I If f has a local maximum at $x = c$ and is differentiable there, then $f'(c) = 0$

II A triangle with sides 4, 5 and 9 exists

III $|a+b| \leq |a| + |b|$ for all real a, b [statements]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

4 Consider the statements P: "a quadrilateral is a square" and Q: "it is a rectangle". Which of the following is true? [larger set]

A P is necessary but not sufficient for Q

B P is sufficient but not necessary for Q

C P is necessary and sufficient for Q

D P is neither necessary nor sufficient for Q

5 $b_1, b_2, b_3, \ldots$ is a geometric progression with common ratio r $(r \neq 0, r \neq \pm 1)$ and S_n is the sum of its first n terms. Which of the following must be true?

I $b_6 + b_8 = 2b_7$
II $b_6\, b_{12} = b_{10}\, b_8$
III $b_8 = b_6\, r^2$

[middle²]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

6 The equation $2x^3 - 6x^2 = c$ has three distinct real solutions. Which of the following is the complete set of possible values of c?

[root band]

A $c < -8$ or $c > 0$ **B** $c > 0$ **C** $-8 < c < 0$
D $c < -8$ **E** $0 < c < 8$ **F** $-8 \leq c \leq 0$

7 How many real solutions does the equation $|x^2 - 5| = 4$ have?

[touch/cross]

A 0 **B** 1 **C** 2 **D** 3 **E** 4

8 Let $f(x) = ax^2 + bx + c$, where $f(x) = 0$ has two distinct real roots whose sum and product are both positive. Which of the following must be true?

I both roots are positive
II $c > 0$
III the vertex has positive x-coordinate

[statements]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

9 $a_1, a_2, a_3, \ldots$ is an arithmetic progression with non-zero common difference and S_n is the sum of its first n terms. Which of the following must be true?

I $a_3 + a_{14} = a_8 + a_9$
II $S_{10} = 2S_5$
III $a_3 + a_9 = 2a_6$

[2×middle]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

10 The function f is twice differentiable with $f''(x) > 0$ for all x in $[4, 5]$. Which of the following must be true?

I the tangent to $y = f(x)$ at $x = \frac{9}{2}$ lies below the curve on $[4, 5]$

II $f(\frac{9}{2}) < f(4)$

III the trapezium rule with one strip overestimates $\int_4^5 f(x)\,dx$ [chord]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

11 Which of the following is a counterexample to the statement "If $x^2 < 4$ then $x < 2$"? [counterexample]

A $x = -1$

B $x = 2$

C $x = 3$

D $x = 4$

E $x = 6$

F none of these is a counterexample

12 Which of the following is a sufficient condition for $\int_{-1}^{3} f(x)\,dx = 0$? [$\bigstar + \triangle$]

A $f(-x) = -f(x)$ for all x

B $f(-1) = -f(3)$

C $f(1 + x) = f(1 - x)$ for all x

D $f(1 + x) = -f(1 - x)$ for all x

E $f(1) = 0$

F $f(-1) = f(3) = 0$

13 Which of the following statements is/are true?

I The coefficient of x^3 in $(1 + x)^5$ is 9

II $|a + b| \leq |a| + |b|$ for all real a, b

III $\int_{-1}^{1} (x^3 + 1x)\,dx = 0$ [statements]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

14 For real numbers x, which of the following describes the condition $x > 1$ in relation to the condition $x^2 > 1$? [larger set]

A necessary but not sufficient

B sufficient but not necessary

C necessary and sufficient

D neither necessary nor sufficient

15 $f(x) = ax^2 + bx + c$ where $a \neq 0$, and $f(3) = f(0)$. Given also that $f(-1) = 0$, which of the following must be true?

I the sum of the roots of $f(x) = 0$ is 3
II $f(0) = 0$
III $f(4) = 0$

[axis + root]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

16 A function f is defined for all real x and satisfies $f(x) + f(2-x) = 0$ and $f(3+x) = f(3-x)$ for every real x. Given that $f(4) = 4$, what is $f(32)$?

[reflect twice]

A -48 **B** -4 **C** 0 **D** 2 **E** 4 **F** 8

17 A function f satisfies $f'(x) = 3x^3 - \frac{3x^2}{2} - 3x + \frac{3}{2}$ for all real x. Which of the following must be true?

I f has a local minimum at $x = 1$
II f has a local maximum at $x = 1$
III the curve $y = f(x)$ has exactly three turning points

[Vieta on f']

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

18 A function f, continuous on the whole real line, satisfies $f(x) + f(2-x) = -2$ for every real x. The maximum value of f is 6, attained only at $x = 4$. What is (the minimum value of f) + (the value of x at which that minimum occurs)?

[range symmetry]

A -30 **B** -10 **C** -8 **D** -2 **E** 10 **F** 100

19 $f(x) = ax^2 + bx + c$ where $a \neq 0$, and $f(4) = f(-5)$. Given also that $f(1) = 0$, which of the following must be true?

I $f(0) = 0$
II $f(4) = f(-5) = 0$
III the sum of the roots of $f(x) = 0$ is -1

[axis + root]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

20 A function f satisfies $f'(x) = \frac{x^3}{2} + \frac{x^2}{4} - \frac{x}{2} - \frac{1}{4}$ for all real x. Which of the following must be true?

I f has a local minimum at $x = -1$
II f has a local maximum at $x = 1$
III f has a local minimum at $x = 1$

[Vieta on f']

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Answer key

Each answer with the circuit that fires and the one-line reflex. Mark, then for every miss write the circuit name next to the question and redo it the NOIR way.

Set 1, Paper 1

Q	Ans	Circuit	Reflex
1	D	root-pair sum	with $u = \cos x$: $2u^2 - (-\frac{5}{6})u - (\frac{1}{3}) = 0$ has roots $u = -\frac{2}{3}, \frac{1}{4}$; a root with $-1 < u \leq 1$ contributes a pair of x-solutions summing to 2π (for $u = 1$ that pair is $x = 0$ and $x = 2\pi$, the trap most people miss), but $u = -1$ contributes only the single solution $x = \pi$: total 4π
2	F	root sum	root sum is $-b/a = \frac{4}{3}$: $k = \frac{4}{3} - (3) - (-4) = \frac{7}{3}$
3	B	root band	stationary values -3 and -11; count the sign pattern: 1
4	B	seasons	four consecutive terms cancel; 2026 terms leave 2: $\sin 10^\circ + \cos 10^\circ$
5	C	interval-max search	$f = 1$ needs x an even multiple of p; the smallest even multiple of p landing in $[7, 15]$, minimised over $p = 1, 2, \ldots$, occurs at $(p, q) = (1, 8)$: $p + q = 9$
6	C	external tangent	the tangent at $x = -3$ has gradient $3t^2 + b$ and y-intercept $-2t^3 = 54$; passing through $(1, 80)$ forces gradient 26, so $b = -1$ and the x-intercept is $\dfrac{2t^3}{3t^2 + b} = -\frac{27}{13}$
7	C	group then subtract	group then subtract: $c\frac{\beta^3 - \alpha^3}{3} + a\frac{\beta^2 - \alpha^2}{2} + b(\beta - \alpha)$
8	D	root-pair sum	with $u = \cos x$: $2u^2 - (\frac{8}{3})u - (-\frac{2}{3}) = 0$ has roots $u = 1, \frac{1}{3}$; a root with $-1 < u \leq 1$ contributes a pair of x-solutions summing to 2π (for $u = 1$ that pair is $x = 0$ and $x = 2\pi$, the trap most people miss), but $u = -1$ contributes only the single solution $x = \pi$: total 4π
9	B	trig symmetry	lines of symmetry pass through the peaks: $4a + \frac{\pi}{4} = \frac{\pi}{2} + n\pi$
10	E	Vieta	$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$: $c = 36$
11	D	Ceva	Ceva: the product of the three ratios is 1
12	E	$\bigstar^4/12$	tangent to a cubic: $S = \frac{\lvert a \rvert}{12}\bigstar^4 = \frac{1875}{2}$
13	E	external tangent	the tangent at $x = 2$ has gradient $3t^2 + b$ and y-intercept $-2t^3 = -16$; passing through $(-1, -20)$ forces gradient 4, so $b = -8$ and the x-intercept is $\dfrac{2t^3}{3t^2 + b} = 4$
14	C	five key points	point symmetry about the inflection: width $\times$ inflection height $= 10 \times 7 = 70$
15	B	piecewise extremum (parameter search)	$h' = g - f$ is odd and always flips sign at $x = 0$ (that extremum never depends on a). On $x \geq 2$, $h' = a(x - 2) - 1x^2$ is a downward parabola peaking at $\frac{a^2}{4c} - aw$: the peak first touches 0 when $a = 4cw = k = 8$, and past that two more sign changes appear on each side by symmetry. $h(4) = -\frac{16}{3}$, so $k + h(4) = \frac{8}{3}$
16	B	circle-chain counting	if circles of radii p, q touch a line on the same side and touch each other, $\frac{1}{\sqrt{w}} = \frac{1}{\sqrt{p}} + \frac{1}{\sqrt{q}}$ for a circle w wedged between them and the line; here $\frac{1}{\sqrt{r_n}} = \frac{1}{\sqrt{4}} + \frac{n}{\sqrt{25}}$, an arithmetic sequence, so $r_n > \frac{1}{16}$ for $n < \frac{35}{2} \approx 17.50$: 17 values
17	D	two-branch cosine rule	clear denominators in $a\cos A = b\cos B$: $a^2(b^2 + c^2 - a^2) = b^2(a^2 + c^2 - b^2)$ factors as $(a - b)(a + b)(a^2 + b^2 - c^2) = 0$, so $a = b$ or $C = 90^\circ$ (never both, since $a + b > 0$). $a = b$ gives area $\frac{13\sqrt{30}}{2}$; $C = 90^\circ$ with $a + b = 17$ gives $ab = 60$, area 30: the larger is $\frac{13\sqrt{30}}{2}$
18	B	mean-shift Diophantine search	shift to the mean $m = S/3$: with $x = 3a - S$ etc., $x + y + z = 0$ and $x^2 + y^2 + z^2 = 9Q - 3S^2 = 906$, i.e. $x^2 + xy + y^2 = 453$ after eliminating $z = -(x + y)$; a short integer search on this ellipse gives a single triple up to order: $(a, b, c) = (-2, 7, 12)$
19	C	iterate and substitute	for $\lvert x \rvert > 2$, $f(x) - x = (x - 2)(x + 1) > 0$ when $x > 2$ (and symmetrically for $x < -2$), so every solution lies in $[-2, 2]$; write $x = 2\cos\theta$, then $f(2\cos\theta) = 2\cos 2\theta$, so the n-fold iterate is $2\cos(2^4\theta)$: solving $\cos(2^4\theta) = \cos\theta$ on $[0, \pi]$ gives $\theta = \frac{2k\pi}{2^4 - 1}$ (8 values) or $\theta = \frac{2k\pi}{2^4 + 1}$ (9 values), overlapping only at $\theta = 0$: $16 = 2^4$ solutions
20	D	divisor-chain recurrence	$f(n) = \sum_{d \mid n, d < n} f(d)$ depends only on the multiset of exponents in n's prime factorisation; a direct count over $n = 1, \ldots, 150$ gives 8 values with $f(n) = 20$

Set 1, Paper 2

Q	Ans	Circuit	Reflex
1	B	compartmentalise and negate	compartmentalise: flip every quantifier, then flip the final claim
2	A	proof audit	the integrand is not defined at $x = 0$, inside the interval; the fundamental theorem does not apply
3	F	statement testing	test each statement on its own; a single counterexample kills a 'for all'
4	B	larger set	draw the two sets; the sufficient condition is the smaller set
5	G	square the middle	square the middle; blocks of a GP are earlier blocks times r^k
6	C	root band	stationary values 0 and -8; the horizontal line $y = c$ must lie strictly between them
7	E	touch or cross	$x^2 = 9$ or $x^2 = 1$; count the non-negative right-hand sides, doubling positive ones
8	F	statement testing	sketch the parabola from the given facts; look for a counterexample before believing a 'must'
9	F	double the middle	equidistant terms have the same sum; $S_n = n\times$ middle term
10	F	midpoint chord	concave up: chords sit above the curve, tangents below, gradient increases
11	F	counterexample	the statement is true: $x^2 < 4$ means $-2 < x < 2$; no counterexample exists
12	D	$\bigstar + \triangle = 2a$	point symmetry about $(1, 0)$, the midpoint of the interval, kills the integral
13	G	statement testing	test each statement on its own; a single counterexample kills a 'for all'
14	B	larger set	$x^2 > 1$ is $x > 1$ or $x < -1$: the larger set
15	F	shared axis, given root	$f(3) = f(0)$ forces the axis of symmetry to $x = \frac{3+0}{2} = \frac{3}{2}$ regardless of a and c; reflecting the given root -1 in that axis gives the other root 4 (equal to it only if $r = \frac{3}{2}$, which is excluded here)
16	B	compose two reflections	point symmetry about $(1, 0)$ plus line symmetry about $x = 3$ compose: shifting by $4 = 2(3-1)$ sends $f(x) \mapsto 2b - f(x)$, and shifting by 8 returns $f(x)$ unchanged. From 4 to 32 is 7 such half-shifts, an odd count, so $f(32) = 2b - f(4) = -4$
17	F	Vieta on f'	sketch the sign of f' across each root: it changes sign at a simple root and holds its sign through a repeated one
18	B	range symmetry	$f(x) + f(2a - x) = 2b$ is point symmetry about $(1, -1)$, so the range is symmetric about -1: minimum $= 2b - M = -8$; the symmetry sends the maximiser $x = 4$ to $x = 2a - c = -2$: $-8 + -2 = -10$
19	D	shared axis, given root	$f(4) = f(-5)$ forces the axis of symmetry to $x = \frac{4+-5}{2} = -\frac{1}{2}$ regardless of a and c; reflecting the given root 1 in that axis gives the other root -2 (equal to it only if $r = -\frac{1}{2}$, which is excluded here)
20	F	Vieta on f'	sketch the sign of f' across each root: it changes sign at a simple root and holds its sign through a repeated one