TMUA9.
MOCK BETA.

One set, Paper 1 and Paper 2, TMUA format.

Free. No NOIR PREMIERE required.
Official time 75 minutes. NOIR target: 15.

TMUA9.

How to use this mock

One set: a Paper 1 (Applications of Mathematical Knowledge) and a Paper 2 (Mathematical Reasoning), twenty multiple-choice questions each, in the TMUA format: pick one letter, no working marked, no calculator.

The point. The official paper gives you 75 minutes. A NOIR-trained student should be done in 15. Every question is one reaction circuit with nothing else in the way, named in cyan at the end of the question, except the last few in each paper, which chain two or three ideas together and are left unnamed on purpose: that is the part of the paper built to feel like a real Q15-20.

The rule. Set the timer for 15 minutes per paper. Stop when it rings, whatever is left is a miss. Mark from the key at the back: every answer comes with the circuit and the one-line reflex.

Verification. Every numeric answer was recomputed from the underlying integral, root, or coordinate geometry by a computer algebra system, independently of the shortcut; every logic answer was checked by brute force where a predicate could be evaluated.

Want more of this. This is one of eight free sets in the TMUA9 Question Bank. The full ten-set TMUA9. Papers, the DELUXE rehearsal, and NOIR PREMIERE go further: `https://tmua9.com/noir-premiere`.

Set 1 Paper 1 Applications of Mathematical Knowledge

TMUA9. PAPERS

20 questions. Official time 75 minutes. NOIR target: 15 minutes. Every question is one reaction circuit, named in the margin. Answer on the grid at the end of the set.

1 How many distinct real roots does the equation $4x^3+30x^2+48x+(-33)=0$ have? [root band]

A 0 **B** 1 **C** 2 **D** 3

2 What is the sum of all solutions of the equation $\cos 2x = \frac{5}{6}\cos x - \frac{1}{2}$ in the interval $0 \le x \le 2\pi$? [root-pair sum]

A π **B** 2π **C** 3π **D** $4\pi-2$ **E** 4π **F** $1+4\pi$

3 The function $f(x)=\cos(\frac{\pi x}{p})$ is considered only on the closed interval $[15,25]$. For natural numbers p and q with $15 \le q \le 25$, f attains its maximum value 1 at $x=q$. What is the least possible value of $p+q$? [$p+q$ search]

A 15 **B** 16 **C** 17 **D** 18 **E** 19 **F** 25

4 The tangent to the curve $y=x^3+bx$ (where b is a constant) at the point where $x=-3$ passes through the point $(-1,19)$, which does not lie on the curve. What is the x-intercept of this tangent? [ext. tangent]

A $-\frac{648}{35}$ **B** $-\frac{162}{35}$ **C** -3
D $-\frac{72}{35}$ **E** $-\frac{54}{35}$ **F** -1

5 The sum of the first n terms of a sequence is $S_n=n^2+4n$. Which of the following is the nth term a_n? [truncate]

A 2 n + 2 **B** 2 n + 5 **C** 4 n + 6 **D** 2 n + 3 **E** 2 n + 4

6 The function f satisfies $f(-5+x)+f(3-x)=0$ for all real x. The graph of $y=f(x)$ has rotational symmetry of order 2 about which point? [$\bigstar+\triangle$]

A $(0,0)$ **B** $(-2,0)$ **C** $(-1,0)$ **D** $(4,0)$ **E** $(-1,1)$ **F** $(1,0)$

7 The tangent to the curve $y=x^3+bx$ (where b is a constant) at the point where $x=-2$ passes through the point $(-4,-24)$, which does not lie on the curve. What is the x-intercept of this tangent? [ext. tangent]

A -4 **B** $-\frac{16}{5}$ **C** $-\frac{13}{5}$
D -2 **E** $-\frac{8}{5}$ **F** $-\frac{4}{5}$

8 The function $f(x)=\cos(\frac{\pi x}{p})$ is considered only on the closed interval $[9,17]$. For natural numbers p and q with $9 \le q \le 17$, f attains its maximum value 1 at $x=q$. What is the least possible value of $p+q$? [$p+q$ search]

A 9 **B** 10 **C** 11 **D** 12 **E** 13 **F** 17

9 A straight line meets $y=2x^4-12x^3+cx^2+dx+e$ where $x=-3$, $x=5$, $x=-2$ and $x=k$. What is k? [root sum]

A -12 **B** -6 **C** 4 **D** 6 **E** 7 **F** 36

10 A geometric progression has sum to infinity 8, and the sum to infinity of the squares of its terms is $\frac{64}{7}$. What is the first term? [middle^2]

A $\frac{8}{7}$ **B** 2 **C** $\frac{8\sqrt{7}}{7}$
D $\frac{7}{2}$ **E** 4 **F** 24

11 What is the value of $\sum_{k=0}^{100}\sin(70+90k)^\circ$? [seasons]

A $-\cos 70^\circ$ **B** $2\sin 70^\circ$
C $\sin 70^\circ - \cos 70^\circ$ **D** $\sin 70^\circ$
E $\sin 70^\circ + \cos 70^\circ$ **F** $\cos 70^\circ$

12 An arithmetic progression has first term a and common difference $d \neq 0$. Its sums satisfy $S_2 = S_5$. Which of the following is true? [$n\times$middle]

A $a = -3d$ **B** $a = 3d$ **C** $a = -\frac{7}{2}d$
D $a = -\frac{5}{2}d$ **E** $a = -4d$ **F** $a = \frac{7}{2}d$

13 A sequence $a_1, a_2, a_3, \ldots$ satisfies $a_1 = 0$ and $a_n + a_{n+1} = 4n + 1$ for every positive integer n. What is a_7?[pair the recurrence]

A 11 **B** 12 **C** 13 **D** 14 **E** 16 **F** 144

14 What is the sum of all solutions of the equation $\cos 2x = \frac{5}{2}\cos x - \frac{7}{4}$ in the interval $0 \le x \le 2\pi$?[root-pair sum]

A π **B** 2π **C** 3π **D** $4\pi - 1$ **E** 4π **F** 16π

15 Functions f and g are defined for all real x by $f(x) = 3x|x|$ and

$$g(x) = \begin{cases} a(x+3) & x < -3 \\ 0 & -3 \le x \le 3 \\ a(x-3) & x > 3 \end{cases}$$

where $a > 0$ is a constant. Let $h(x) = \int_0^x (g(u) - f(u))\,du$. The largest value of a for which h attains a local maximum or a local minimum value at exactly one value of x is k. When $a = k$, what is $k + h(6)$? [piecewise extremum]

A -54 **B** -18 **C** -6 **D** 36 **E** 39 **F** 90

16 Two circles, of radii 4 and 1, touch one another and both touch a common straight line ℓ, on the same side of ℓ. Circles $C_1, C_2, C_3, \ldots$ are then inscribed in the successive gaps: each C_n touches ℓ, touches the circle of radius 4, and touches C_{n-1} (with C_0 the circle of radius 1). For how many values of n does C_n have radius greater than $\frac{1}{100}$? [circle chain]

A 15 **B** 17 **C** 18 **D** 19 **E** 51 **F** 68

17 Triangle ABC has $BC = a$, $CA = b$, $AB = c = 13$, and $a\cos A = b\cos B$. Given also that $a + b = 15$, what is the largest possible area of the triangle? [two-branch cosine rule]

A $\frac{13\sqrt{14}}{6}$ **B** 14
C $\frac{13\sqrt{14}}{2}$ **D** $\frac{28+13\sqrt{14}}{2}$
E $\frac{195}{2}$ **F** $78\sqrt{14}$

18 Three distinct integers $a < b < c$ satisfy

$$a + b + c = -7, \qquad a^2 + b^2 + c^2 = 419.$$

What is the middle of the three? [mean-shift Diophantine]

A -20 **B** -15 **C** -7 **D** -5 **E** -4 **F** 13

19 $f(x) = x^2 - 2$. How many real solutions does $\underbrace{f(f(\cdots f}_{4}(x)\cdots)) = x$ have? [iterate + substitution]

A 4 **B** 8 **C** 15 **D** 16 **E** 17 **F** 48

20 For each integer $n \ge 1$ let $f(n)$ be the number of lists of *distinct* positive integers which begin with 1, end with n, and in which every term except the last divides the term after it. (For example $f(1) = 1$, $f(2) = 1$ and $f(6) = 3$, the three lists being $1, 6$ and $1, 2, 6$ and $1, 3, 6$.) How many integers n with $1 < n \le 100$ satisfy $f(n) = 20$? [divisor chain]

A 4 **B** 5 **C** 6 **D** 15 **E** 25 **F** 30

Set 1 Paper 2 Mathematical Reasoning

TMUA9. PAPERS

20 questions. Official time 75 minutes. NOIR target: 15 minutes. Every question is one reaction circuit, named in the margin. Answer on the grid at the end of the set.

1 Which of the following is a counterexample to the statement "If n is a positive integer then n^2+n+11 is prime"? [counterexample]

A $n=3$ **B** $n=5$ **C** $n=10$ **D** $n=23$ **E** $n=29$

2 $ABCD$ is a cyclic quadrilateral with $\angle ABC = 100°$. Which of the following must be true?

I $\angle ADC = 100°$
II $\angle ADC = 80°$
III the exterior angle at D equals $100°$ [cyclic]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

3 Which of the following is the negation of the statement "For every real number x, $x^2+3>3x$."? [negate]

A There is a real number x for which $x^2+3 \leq 3x$.
B For every real number x, $x^2+3<3x$.
C There is a real number x for which $x^2+3<3x$.
D There is a real number x for which $x^2+3>3x$.
E There is no real number x for which $x^2+3>3x$.
F For every real number x, $x^2+3 \leq 3x$.

4 Which of the following is the negation of the statement "Some student scored at least 80 marks."? [negate]

A Some student scored fewer than 80 marks.
B No student scored more than 80 marks.
C Some student scored more than 80 marks.
D Every student scored at least 80 marks.
E Every student scored fewer than 80 marks.
F Every student scored at most 80 marks.

5 The function f is twice differentiable with $f''(x)>0$ for all x in $[1,2]$. Which of the following must be true?

I $f'(1)<f'(2)$
II the tangent to $y=f(x)$ at $x=\frac{3}{2}$ lies below the curve on $[1,2]$
III $f(1)<f(2)$ [chord]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

6 Consider the statements P: "$x^2 > 9$" and Q: "$x > 3$". Which of the following is true? [larger set]

A P is necessary but not sufficient for Q

B P is sufficient but not necessary for Q

C P is necessary and sufficient for Q

D P is neither necessary nor sufficient for Q

7 Which of the following statements is/are true?

I $\cos^2 x - \sin^2 x = \cos 2x$ for all real x
II Every quartic polynomial has at least one real root
III The coefficient of x^1 in $(1+x)^4$ is 4 [statements]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

8 How many distinct real solutions does the equation $(x-1)^2(x-4) = 5$ have? [root band]

A 0 **B** 1 **C** 2 **D** 3

9 Which of the following is the inverse of the statement "if $x > 5$ then $x^2 > 25$"? [converse]

A if $x^2 > 25$ then $x \leq 5$

B if $x^2 \leq 25$ then $x > 5$

C if $x^2 > 25$ then $x > 5$

D if $x > 5$ then $x^2 > 25$

E if $x \leq 5$ then $x^2 \leq 25$

F if $x^2 \leq 25$ then $x \leq 5$

10 Which of the following statements is/are true?

I The equation $x^3 - 3x + 4 = 0$ has three distinct real roots
II $x^2 - 6x + 9 > 0$ for all real x
III The sequence $a_n = 5^n$ is arithmetic [statements]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

11 For the curve $y = (x+2)(x+3)^2(x-4)^2$, which of the following is true when x is a large negative number? [ends]

A y is positive and $|y|$ is large

B y is negative and $|y|$ is large

C y is close to 0

D y is positive and close to 1

E y alternates in sign

12 $b_1, b_2, b_3, \ldots$ is a geometric progression with common ratio r ($r \neq 0, r \neq \pm 1$) and S_n is the sum of its first n terms. Which of the following must be true?

I if $|r| < 1$ then the sum to infinity is $\dfrac{b_1}{1-r}$
II $b_5 = b_1 r^4$
III $\dfrac{S_4}{S_2} = 1 + r^2$ [middle²]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

13 A function f satisfies $f(x) + f(-6-x) = -8$ for every real x. Given that $f(1) = -4$, what is $f(-7)$? [range symmetry]

A -16 **B** -12 **C** -6 **D** -4 **E** -3 **F** 4

14 Let $f(x) = ax^2 + bx + c$ with $a > 0$ and $f(1) < 0$. Which of the following must be true?
I the roots of $f(x) = 0$ lie on either side of $x = 1$
II $f(0) < 0$
III $b^2 > 4ac$ [statements]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

15 $f(x) = ax^2 + bx + c$ where $a \neq 0$, and $f(-2) = f(5)$. Given also that $f(6) = 0$, which of the following must be true?
I $f(-2) = f(5) = 0$
II the sum of the roots of $f(x) = 0$ is 3
III $f(\frac{3}{2})$ is the least value of f if $a > 0$, and the greatest if $a < 0$ [axis + root]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

16 A function f is defined for all real x and satisfies $f(x) + f(6-x) = 6$ and $f(-3+x) = f(-3-x)$ for every real x. Given that $f(-2) = 2$, what is $f(-62)$? [reflect twice]

A -10 **B** 2 **C** 4 **D** 6 **E** 8 **F** 12

17 A function f satisfies $f'(x) = \frac{x^3}{2} - \frac{3x^2}{2} - \frac{x}{2} + \frac{3}{2}$ for all real x. Which of the following must be true?

I f has a local minimum at $x = -1$

II f has a local maximum at $x = -1$

III the equation $f'(x) = 0$ has a root strictly between -1 and 1 [Vieta on f']

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

18 A function f, continuous on the whole real line, satisfies $f(x) + f(6 - x) = -4$ for every real x. The maximum value of f is 5, attained only at $x = 1$. What is (the minimum value of f) + (the value of x at which that minimum occurs)? [range symmetry]

A -48 **B** -9 **C** -4 **D** -2 **E** 5 **F** 6

19 A function f, continuous on the whole real line, satisfies $f(x) + f(-x) = 4$ for every real x. The maximum value of f is 5, attained only at $x = 1$. What is (the minimum value of f) + (the value of x at which that minimum occurs)? [range symmetry]

A -12 **B** -2 **C** -1 **D** 2 **E** 4 **F** 6

20 A function f, continuous on the whole real line, satisfies $f(x) + f(6 - x) = 0$ for every real x. The maximum value of f is 10, attained only at $x = -2$. What is (the minimum value of f) + (the value of x at which that minimum occurs)? [range symmetry]

A -10 **B** -8 **C** -2 **D** -1 **E** 4 **F** 8

Answer key

Each answer with the circuit that fires and the one-line reflex. Mark, then for every miss write the circuit name next to the question and redo it the NOIR way.

Set 1, Paper 1

Q	Ans	Circuit	Reflex
1	B	root band	stationary values -1 and -55; count the sign pattern: 1
2	E	root-pair sum	with $u = \cos x$: $2u^2 - (\frac{5}{6})u - (\frac{1}{2}) = 0$ has roots $u = -\frac{1}{3}, \frac{3}{4}$; a root with $-1 < u \leq 1$ contributes a pair of x-solutions summing to 2π (for $u = 1$ that pair is $x = 0$ and $x = 2\pi$, the trap most people miss), but $u = -1$ contributes only the single solution $x = \pi$: total 4π
3	C	interval-max search	$f = 1$ needs x an even multiple of p; the smallest even multiple of p landing in $[15, 25]$, minimised over $p = 1, 2, \ldots$, occurs at $(p, q) = (1, 16)$: $p + q = 17$
4	E	external tangent	the tangent at $x = -3$ has gradient $3t^2 + b$ and y-intercept $-2t^3 = 54$; passing through $(-1, 19)$ forces gradient 35, so $b = 8$ and the x-intercept is $\dfrac{2t^3}{3t^2 + b} = -\frac{54}{35}$
5	D	truncate and integrate	$a_n = S_n - S_{n-1}$, or differentiate the leading term and fix with $a_1 = S_1$
6	C	$\bigstar + \triangle = 2a$	arguments sum to $2a = -2$, values sum to $2b = 0$: $(-1, 0)$
7	E	external tangent	the tangent at $x = -2$ has gradient $3t^2 + b$ and y-intercept $-2t^3 = 16$; passing through $(-4, -24)$ forces gradient 10, so $b = -2$ and the x-intercept is $\dfrac{2t^3}{3t^2 + b} = -\frac{8}{5}$
8	C	interval-max search	$f = 1$ needs x an even multiple of p; the smallest even multiple of p landing in $[9, 17]$, minimised over $p = 1, 2, \ldots$, occurs at $(p, q) = (1, 10)$: $p + q = 11$
9	D	root sum	quartic root sum $-b/a = 6$: $k = 6$
10	B	square the middle	$\frac{S_\infty^2}{S_\infty^{(2)}} = \frac{1+r}{1-r}$ gives $r = \frac{3}{4}$, then $a = 2$
11	D	seasons	four consecutive terms cancel; 101 terms leave 1: $\sin 70°$
12	A	$S_n = n \times$middle	terms 3 to 5 sum to zero, so their middle term $a + \frac{6}{2}d = 0$
13	B	paired recurrence	unwind $a_{n+1} = (4n + 1) - a_n$ from $a_1 = 0$ down to $a_7 = 12$
14	E	root-pair sum	with $u = \cos x$: $2u^2 - (\frac{5}{2})u - (-\frac{3}{4}) = 0$ has roots $u = \frac{1}{2}, \frac{3}{4}$; a root with $-1 < u \leq 1$ contributes a pair of x-solutions summing to 2π (for $u = 1$ that pair is $x = 0$ and $x = 2\pi$, the trap most people miss), but $u = -1$ contributes only the single solution $x = \pi$: total 4π
15	B	piecewise extremum (parameter search)	$h' = g - f$ is odd and always flips sign at $x = 0$ (that extremum never depends on a). On $x \geq 3$, $h' = a(x-3) - 3x^2$ is a downward parabola peaking at $\frac{a^2}{4c} - aw$: the peak first touches 0 when $a = 4cw = k = 36$, and past that two more sign changes appear on each side by symmetry. $h(6) = -54$, so $k + h(6) = -18$
16	B	circle-chain counting	if circles of radii p, q touch a line on the same side and touch each other, $\frac{1}{\sqrt{w}} = \frac{1}{\sqrt{p}} + \frac{1}{\sqrt{q}}$ for a circle w wedged between them and the line; here $\frac{1}{\sqrt{r_n}} = \frac{1}{\sqrt{1}} + \frac{n}{\sqrt{4}}$, an arithmetic sequence, so $r_n > \frac{1}{100}$ for $n < 18 \approx 18.00$: 17 values
17	C	two-branch cosine rule	clear denominators in $a\cos A = b\cos B$: $a^2(b^2 + c^2 - a^2) = b^2(a^2 + c^2 - b^2)$ factors as $(a - b)(a + b)(a^2 + b^2 - c^2) = 0$, so $a = b$ or $C = 90°$ (never both, since $a + b > 0$). $a = b$ gives area $\frac{13\sqrt{14}}{2}$; $C = 90°$ with $a + b = 15$ gives $ab = 28$, area 14: the larger is $\frac{13\sqrt{14}}{2}$
18	D	mean-shift Diophantine search	shift to the mean $m = S/3$: with $x = 3a - S$ etc., $x + y + z = 0$ and $x^2 + y^2 + z^2 = 9Q - 3S^2 = 3624$, i.e. $x^2 + xy + y^2 = 1812$ after eliminating $z = -(x + y)$; a short integer search on this ellipse gives a single triple up to order: $(a, b, c) = (-15, -5, 13)$
19	D	iterate and substitute	for $\lvert x\rvert > 2$, $f(x) - x = (x - 2)(x + 1) > 0$ when $x > 2$ (and symmetrically for $x < -2$), so every solution lies in $[-2, 2]$; write $x = 2\cos\theta$, then $f(2\cos\theta) = 2\cos 2\theta$, so the n-fold iterate is $2\cos(2^4\theta)$: solving $\cos(2^4\theta) = \cos\theta$ on $[0, \pi]$ gives $\theta = \frac{2k\pi}{2^4 - 1}$ (8 values) or $\theta = \frac{2k\pi}{2^4 + 1}$ (9 values), overlapping only at $\theta = 0$: $16 = 2^4$ solutions
20	B	divisor-chain recurrence	$f(n) = \sum_{d \mid n,\, d < n} f(d)$ depends only on the multiset of exponents in n's prime factorisation; a direct count over $n = 1, \ldots, 100$ gives 5 values with $f(n) = 20$

Set 1, Paper 2

Q	Ans	Circuit	Reflex
1	C	counterexample	$n = 10$ gives 121, which is not prime
2	G	cyclic quadrilateral	opposite angles are supplementary; exterior angle equals the opposite interior angle; Ptolemy
3	A	compartmentalise and negate	compartmentalise: flip every quantifier, then flip the final claim
4	E	compartmentalise and negate	compartmentalise: flip every quantifier, then flip the final claim
5	E	midpoint chord	concave up: chords sit above the curve, tangents below, gradient increases
6	A	larger set	draw the two sets; the sufficient condition is the smaller set
7	F	statement testing	test each statement on its own; a single counterexample kills a 'for all'
8	B	root band	sketch the cubic (touch at one root, cross at the other) and slide the horizontal line
9	E	converse / contrapositive	inverse: if $x \leq 5$ then $x^2 \leq 25$; it is false here
10	A	statement testing	test each statement on its own; a single counterexample kills a 'for all'
11	B	end behaviour	degree 5 with positive leading coefficient: sign at $-\infty$ is $(-1)^5$
12	H	square the middle	square the middle; blocks of a GP are earlier blocks times r^k
13	D	range symmetry	substitute $x = 1$: $f(1) + f(-7) = -8$, so $f(-7) = -8 - (-4) = -4$
14	F	statement testing	sketch the parabola from the given facts; look for a counterexample before believing a 'must'
15	G	shared axis, given root	$f(-2) = f(5)$ forces the axis of symmetry to $x = \frac{-2+5}{2} = \frac{3}{2}$ regardless of a and c; reflecting the given root 6 in that axis gives the other root -3 (equal to it only if $r = \frac{3}{2}$, which is excluded here)
16	C	compose two reflections	point symmetry about $(3, 3)$ plus line symmetry about $x = -3$ compose: shifting by $-12 = 2(-3-3)$ sends $f(x) \mapsto 2b - f(x)$, and shifting by -24 returns $f(x)$ unchanged. From -2 to -62 is 5 such half-shifts, an odd count, so $f(-62) = 2b - f(-2) = 4$
17	B	Vieta on f'	sketch the sign of f' across each root: it changes sign at a simple root and holds its sign through a repeated one
18	C	range symmetry	$f(x) + f(2a - x) = 2b$ is point symmetry about $(3, -2)$, so the range is symmetric about -2: minimum $= 2b - M = -9$; the symmetry sends the maximiser $x = 1$ to $x = 2a - c = 5$: $-9 + 5 = -4$
19	B	range symmetry	$f(x) + f(2a - x) = 2b$ is point symmetry about $(0, 2)$, so the range is symmetric about 2: minimum $= 2b - M = -1$; the symmetry sends the maximiser $x = 1$ to $x = 2a - c = -1$: $-1 + -1 = -2$
20	C	range symmetry	$f(x) + f(2a - x) = 2b$ is point symmetry about $(3, 0)$, so the range is symmetric about 0: minimum $= 2b - M = -10$; the symmetry sends the maximiser $x = -2$ to $x = 2a - c = 8$: $-10 + 8 = -2$