TMUA9.
MOCK DELTA.

One set, Paper 1 and Paper 2, TMUA format.

Free. No NOIR PREMIERE required.
Official time 75 minutes. NOIR target: 15.

TMUA9.

How to use this mock

One set: a Paper 1 (Applications of Mathematical Knowledge) and a Paper 2 (Mathematical Reasoning), twenty multiple-choice questions each, in the TMUA format: pick one letter, no working marked, no calculator.

The point. The official paper gives you 75 minutes. A NOIR-trained student should be done in 15. Every question is one reaction circuit with nothing else in the way, named in cyan at the end of the question, except the last few in each paper, which chain two or three ideas together and are left unnamed on purpose: that is the part of the paper built to feel like a real Q15-20.

The rule. Set the timer for 15 minutes per paper. Stop when it rings, whatever is left is a miss. Mark from the key at the back: every answer comes with the circuit and the one-line reflex.

Verification. Every numeric answer was recomputed from the underlying integral, root, or coordinate geometry by a computer algebra system, independently of the shortcut; every logic answer was checked by brute force where a predicate could be evaluated.

Want more of this. This is one of eight free sets in the TMUA9 Question Bank. The full ten-set TMUA9. Papers, the DELUXE rehearsal, and NOIR PREMIERE go further: `https://tmua9.com/noir-premiere`.

Set 1 Paper 1 Applications of Mathematical Knowledge

TMUA9. PAPERS

20 questions. Official time 75 minutes. NOIR target: 15 minutes. Every question is one reaction circuit, named in the margin. Answer on the grid at the end of the set.

1 The function f satisfies $f(-5+x)+f(8-x)=8$ for all real x. The graph of $y=f(x)$ has rotational symmetry of order 2 about which point? [$\bigstar+\triangle$]

A $(\frac{3}{2},-4)$ **B** $(3,4)$ **C** $(\frac{3}{2},8)$
D $(\frac{13}{2},4)$ **E** $(\frac{3}{2},0)$ **F** $(\frac{3}{2},4)$

2 How many distinct real roots does the equation $2x^3+6x^2+(0)=0$ have? [root band]

A 0 **B** 1 **C** 2 **D** 3

3 What is the smallest positive value of a for which the line $x=a$ is a line of symmetry of the curve $y=\sin(x-\frac{\pi}{3})$? [trig sym]

A $\frac{\pi}{3}$ **B** $\frac{5\pi}{12}$ **C** $\frac{5\pi}{6}$
D $\frac{6+5\pi}{6}$ **E** $\frac{5\pi}{4}$ **F** $\frac{11\pi}{6}$

4 P lies on the circle $(x-5)^2+y^2=25$ and Q lies on the circle $(x-13)^2+(y-15)^2=1$. What is the least possible length of PQ? [centres]

A $\frac{17}{2}$ **B** 11 **C** 12 **D** 16 **E** 17 **F** 23

5 What is the value of $\sum_{k=0}^{2025}\cos(10+90k)^\circ$? [seasons]

A $\sin 10^\circ$ **B** $\cos 10^\circ-\sin 10^\circ$
C $-\cos 10^\circ$ **D** $2\sin 10^\circ$
E $\cos 10^\circ$ **F** $\sin 10^\circ-\cos 10^\circ$

6 What is the term independent of x in the expansion of $(x^1-\frac{1}{x^2})^6$? [multinomial]

A -15 **B** 0 **C** 6 **D** 14 **E** 15 **F** 20

7 The tangent to the curve $y=x^3+bx$ (where b is a constant) at the point where $x=1$ passes through the point $(-3,7)$, which does not lie on the curve. What is the x-intercept of this tangent? [ext. tangent]

A -3 **B** $-\frac{4}{3}$ **C** -1 **D** $-\frac{2}{3}$ **E** $\frac{1}{3}$ **F** 1

8 The function $f(x)=\cos(\frac{\pi x}{p})$ is considered only on the closed interval $[19,25]$. For natural numbers p and q with $19\le q\le 25$, f attains its maximum value 1 at $x=q$. What is the least possible value of $p+q$? [$p+q$ search]

A 19 **B** 20 **C** 21 **D** 22 **E** 23 **F** 25

9 What is the area of the finite region enclosed between the curve $y=2x^2-4x-5$ and the line $y=6x-5$? [$\sqrt{D}/|a|$]

A $\frac{625}{18}$ **B** $\frac{119}{3}$ **C** $\frac{125}{3}$
D 50 **E** $\frac{125}{2}$ **F** $\frac{15625}{9}$

10 The tangent to a parabola at P has gradient 1959 and the tangent at Q has gradient 1955. What is the gradient of the chord PQ? [$S{:}T$]

A 2 **B** 1956 **C** $\sqrt{3829845}$ **D** 1957 **E** 3914 **F** 7828

11 A cubic curve $y = x^3 + ax^2 + bx$ (where a and b are constants) has a local maximum at $x = 0$ and a local minimum at $x = 1$. What is ab? [Vieta on f']

A -2 **B** -1 **C** 0 **D** 1 **E** 2

12 A geometric progression has common ratio 2. What is $\frac{S_6}{S_3}$? [blocks]

A 2 **B** 5 **C** 8 **D** 9 **E** 16 **F** 65

13 A sequence $a_1, a_2, a_3, \ldots$ satisfies $a_1 = 0$ and $a_n + a_{n+1} = 2n - 1$ for every positive integer n. What is a_{10}? [pair the recurrence]

A 3 **B** 6 **C** 8 **D** 9 **E** 10 **F** 11

14 A straight line is a tangent to $y = 3x^3 + 9x^2 + cx + d$ at the point where $x = 4$ and meets the curve again where $x = k$. What is k? [root sum]

A -11 **B** -10 **C** -7 **D** -5 **E** 5 **F** 11

15 Functions f and g are defined for all real x by $f(x) = 2x|x|$ and

$$g(x) = \begin{cases} a(x+3) & x < -3 \\ 0 & -3 \le x \le 3 \\ a(x-3) & x > 3 \end{cases}$$

where $a > 0$ is a constant. Let $h(x) = \int_0^x (g(u) - f(u))\,du$. The largest value of a for which h attains a local maximum or a local minimum value at exactly one value of x is k. When $a = k$, what is $k + h(6)$? [piecewise extremum]

A -36 **B** -12 **C** -8 **D** 24 **E** 27 **F** 60

16 Two circles, of radii 9 and 4, touch one another and both touch a common straight line ℓ, on the same side of ℓ. Circles $C_1, C_2, C_3, \ldots$ are then inscribed in the successive gaps: each C_n touches ℓ, touches the circle of radius 9, and touches C_{n-1} (with C_0 the circle of radius 4). For how many values of n does C_n have radius greater than $\frac{1}{100}$? [circle chain]

A 7 **B** 21 **C** 28 **D** 29 **E** 168 **F** 784

17 Triangle ABC has $BC = a$, $CA = b$, $AB = c = 6$, and $a\cos A = b\cos B$. Given also that $a + b = 8$, what is the largest possible area of the triangle? [two-branch cosine rule]

A $\sqrt{7}$ **B** 7 **C** $3\sqrt{7}$
D $7 + 3\sqrt{7}$ **E** $9\sqrt{7}$ **F** 24

18 Three distinct integers $a < b < c$ satisfy

$$a + b + c = 23, \qquad a^2 + b^2 + c^2 = 291.$$

What is the largest of the three? [mean-shift Diophantine]

A -1 **B** 11 **C** 13 **D** 15 **E** 23 **F** 39

19 $f(x) = x^2 - 2$. How many real solutions does $\underbrace{f(f(\cdots f}_{3}(x)\cdots)) = x$ have? [iterate + substitution]

A 4 **B** 6 **C** 7 **D** 8 **E** 9 **F** 48

20 For each integer $n \ge 1$ let $f(n)$ be the number of lists of *distinct* positive integers which begin with 1, end with n, and in which every term except the last divides the term after it. (For example $f(1) = 1$, $f(2) = 1$ and $f(6) = 3$, the three lists being 1, 6 and 1, 2, 6 and 1, 3, 6.) How many integers n with $1 < n \le 150$ satisfy $f(n) = 44$? [divisor chain]

A 5 **B** 6 **C** 7 **D** 8 **E** 9 **F** 84

Set 1 Paper 2 Mathematical Reasoning

TMUA9. PAPERS

20 questions. Official time 75 minutes. NOIR target: 15 minutes. Every question is one reaction circuit, named in the margin. Answer on the grid at the end of the set.

1 A, B and P lie on a circle with centre O, and $\angle APB = 120°$. Which of the following must be true?

I the reflex angle AOB is $240°$
II $\angle AQB = 60°$ for every point Q on the other arc
III $\angle AQB = 120°$ for every point Q on the same arc as P [inscribed]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

2 Which of the following is the contrapositive of the statement "if $x > 2$ then $x^2 > 4$"? [converse]

A if $x > 2$ then $x^2 > 4$

B if $x^2 \leq 4$ then $x \leq 2$

C if $x^2 \leq 4$ then $x > 2$

D if $x^2 > 4$ then $x > 2$

E if $x \leq 2$ then $x^2 \leq 4$

F if $x^2 > 4$ then $x \leq 2$

3 Claim: if $\sin\theta = \sin\phi$ then $\theta = \phi$. Consider the following argument.

1 $\sin\theta - \sin\phi = 2\cos\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$
2 if this is 0 then $\sin\frac{\theta-\phi}{2} = 0$
3 so $\theta - \phi = 0$
4 hence $\theta = \phi$

Which of the following best describes the argument? [proof]

A The argument is incorrect: the first error is in line 2

B The argument is incorrect: the first error is in line 1

C The argument is incorrect: the first error is in line 3

D The argument is incorrect: the first error is in line 4

E The argument is correct

4 How many solutions does the equation $\sin(2x) = \frac{3}{4}$ have in the interval $0 \leq x \leq 2\pi$? [seasons]

A 2 **B** 3 **C** 4 **D** 5 **E** 8

5 $p(x)$ is a polynomial of degree 5 with positive leading coefficient, and $p(0)$ is negative. Which of the following statements must be true?

I $p(x) = 0$ has at least two real roots
II $p(x)$ has exactly 4 turning points
III $p(x) = 0$ has an even number of real roots (counted with multiplicity) [ends]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

6 x and y are positive real numbers with $xy = 36$. Which of the following must be true?

I $x + y \leq 12$
II $x + y = 12$
III $x^2 + y^2 \geq 72$ [AM-GM]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

7 Which of the following is the negation of the statement "For every real number x, $x^2 + 2 > 1x$."? [negate]

A There is a real number x for which $x^2 + 2 > 1x$.

B For every real number x, $x^2 + 2 \leq 1x$.

C There is a real number x for which $x^2 + 2 < 1x$.

D There is no real number x for which $x^2 + 2 > 1x$.

E There is a real number x for which $x^2 + 2 \leq 1x$.

F For every real number x, $x^2 + 2 < 1x$.

8 Consider the statements P: "a triangle is equilateral" and Q: "it is isosceles". Which of the following is true? [larger set]

A P is necessary but not sufficient for Q

B P is sufficient but not necessary for Q

C P is necessary and sufficient for Q

D P is neither necessary nor sufficient for Q

9 Which of the following conditions is/are sufficient for the graph of $y = f(x)$ to have the line $x = 4$ as an axis of symmetry?

I $f(4 + x) + f(4 - x) = 0$ for all x
II $f(x) = f(8 + x)$ for all x
III $f(4 - x) = f(4 + x)$ for all x [$\bigstar + \triangle$]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

10 Let $f(x) = ax^2 + bx + c$ with $a > 0$ and $f(-1) < 0$. Which of the following must be true?

I $f(0) > 0$

II f has a minimum value less than 0

III the roots of $f(x) = 0$ lie on either side of $x = -1$ [statements]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

11 The function f is twice differentiable with $f''(x) > 0$ for all x in $[3, 5]$. Which of the following must be true?

I $f''(x) < 0$ for all x in $[3, 5]$

II the trapezium rule with one strip overestimates $\int_3^5 f(x)\,\mathrm{d}x$

III $\dfrac{f(3) + f(5)}{2} > f(4)$ [chord]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

12 Which of the following is the negation of the statement "Some student scored at least 60 marks."? [negate]

A No student scored more than 60 marks.

B Every student scored at most 60 marks.

C Every student scored fewer than 60 marks.

D Some student scored more than 60 marks.

E Some student scored fewer than 60 marks.

F Every student scored at least 60 marks.

13 How many times does the graph of $y = x^3(x-1)^3(x-4)$ cross the x-axis? [touch/cross]

A 0 **B** 1 **C** 2 **D** 3 **E** 4

14 Which of the following is a sufficient condition for $\int_{-1}^5 f(x)\,\mathrm{d}x = 0$? [★ + △]

A $f(2) = 0$

B $f(2 + x) = f(2 - x)$ for all x

C $f(-1) = -f(5)$

D $f(-x) = -f(x)$ for all x

E $f(2 + x) = -f(2 - x)$ for all x

F $f(-1) = f(5) = 0$

15 $f(x) = ax^2 + bx + c$ where $a \neq 0$, and $f(3) = f(-4)$. Given also that $f(-5) = 0$, which of the following must be true?

I the sum of the roots of $f(x) = 0$ is -1
II $f(3) = f(-4) = 0$
III $f(0) = 0$

[axis + root]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

16 A function f is defined for all real x and satisfies $f(x) + f(-4 - x) = 0$ and $f(1 + x) = f(1 - x)$ for every real x. Given that $f(-3) = 3$, what is $f(-21)$?

[reflect twice]

A -4 **B** -3 **C** -2 **D** 0 **E** 3 **F** 9

17 A function f satisfies $f'(x) = x^3 + 2x^2 - x - 2$ for all real x. Which of the following must be true?

I the curve $y = f(x)$ has exactly three turning points
II f has a local maximum at $x = 1$
III the equation $f'(x) = 0$ has a root strictly between -1 and 1

[Vieta on f']

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

18 A function f, continuous on the whole real line, satisfies $f(x) + f(6 - x) = 6$ for every real x. The maximum value of f is 6, attained only at $x = 0$. What is (the minimum value of f) + (the value of x at which that minimum occurs)?

[range symmetry]

A -6 **B** 0 **C** 5 **D** 6 **E** 8 **F** 9

19 A function f satisfies $f'(x) = 3x^3 + 9x^2 - 3x - 9$ for all real x. Which of the following must be true?

I the curve $y = f(x)$ has exactly three turning points
II f has a local minimum at $x = -1$
III f has a local maximum at $x = 1$

[Vieta on f']

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

20 A function f satisfies $f'(x) = \frac{x^3}{2} + \frac{3x^2}{2} - \frac{x}{2} - \frac{3}{2}$ for all real x. Which of the following must be true?

I f has a local minimum at $x = 1$
II f has a local maximum at $x = -1$
III f has a local maximum at $x = 1$

[Vieta on f']

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Answer key

Each answer with the circuit that fires and the one-line reflex. Mark, then for every miss write the circuit name next to the question and redo it the NOIR way.

Set 1, Paper 1

Q	Ans	Circuit	Reflex
1	F	$\bigstar + \triangle = 2a$	arguments sum to $2a = 3$, values sum to $2b = 8$: $(\frac{3}{2}, 4)$
2	C	root band	stationary values 8 and 0; count the sign pattern: 2
3	C	trig symmetry	lines of symmetry pass through the peaks: $1a - \frac{\pi}{3} = \frac{\pi}{2} + n\pi$
4	B	line of centres	nearest points lie on the line of centres: $d - R - r$ with $d = 17$
5	B	seasons	four consecutive terms cancel; 2026 terms leave 2: $\cos 10^\circ - \sin 10^\circ$
6	E	multinomial	$p(n-k) = qk$ gives $k = 2$: $\binom{6}{2}(-1)^2 = 15$
7	D	external tangent	the tangent at $x = 1$ has gradient $3t^2 + b$ and y-intercept $-2t^3 = -2$; passing through $(-3, 7)$ forces gradient -3, so $b = -6$ and the x-intercept is $\dfrac{2t^3}{3t^2 + b} = -\frac{2}{3}$
8	C	interval-max search	$f = 1$ needs x an even multiple of p; the smallest even multiple of p landing in $[19, 25]$, minimised over $p = 1, 2, \ldots$, occurs at $(p, q) = (1, 20)$: $p + q = 21$
9	C	$\bigstar = \sqrt{D}/\lvert a\rvert$	$D = 100$, $\bigstar = \sqrt{D}/\lvert a\rvert = 5$, $S = \frac{\lvert a\rvert}{6}\bigstar^3 = \frac{125}{3}$
10	D	$S : T = 2 : 1$	chord gradient is the mean of the tangent gradients: 1957
11	C	Vieta on f'	$f'(x) = 3x^2 - 3x$ must have roots $0, 1$: Vieta on the derivative gives $a = -\frac{3}{2}$, $b = 0$
12	D	GP blocks	the second block is the first block times r^3: $1 + r^3 = 9$
13	D	paired recurrence	unwind $a_{n+1} = (2n - 1) - a_n$ from $a_1 = 0$ down to $a_{10} = 9$
14	A	root sum	tangency counts twice: $k = -3 - 2(4) = -11$
15	B	piecewise extremum (parameter search)	$h' = g - f$ is odd and always flips sign at $x = 0$ (that extremum never depends on a). On $x \geq 3$, $h' = a(x - 3) - 2x^2$ is a downward parabola peaking at $\frac{a^2}{4c} - aw$: the peak first touches 0 when $a = 4cw = k = 24$, and past that two more sign changes appear on each side by symmetry. $h(6) = -36$, so $k + h(6) = -12$
16	C	circle-chain counting	if circles of radii p, q touch a line on the same side and touch each other, $\frac{1}{\sqrt{w}} = \frac{1}{\sqrt{p}} + \frac{1}{\sqrt{q}}$ for a circle w wedged between them and the line; here $\frac{1}{\sqrt{r_n}} = \frac{1}{\sqrt{4}} + \frac{n}{\sqrt{9}}$, an arithmetic sequence, so $r_n > \frac{1}{100}$ for $n < \frac{57}{2} \approx 28.50$: 28 values
17	C	two-branch cosine rule	clear denominators in $a\cos A = b\cos B$: $a^2(b^2 + c^2 - a^2) = b^2(a^2 + c^2 - b^2)$ factors as $(a - b)(a + b)(a^2 + b^2 - c^2) = 0$, so $a = b$ or $C = 90^\circ$ (never both, since $a + b > 0$). $a = b$ gives area $3\sqrt{7}$; $C = 90^\circ$ with $a + b = 8$ gives $ab = 14$, area 7: the larger is $3\sqrt{7}$
18	C	mean-shift Diophantine search	shift to the mean $m = S/3$: with $x = 3a - S$ etc., $x + y + z = 0$ and $x^2 + y^2 + z^2 = 9Q - 3S^2 = 1032$, i.e. $x^2 + xy + y^2 = 516$ after eliminating $z = -(x + y)$; a short integer search on this ellipse gives a single triple up to order: $(a, b, c) = (-1, 11, 13)$
19	D	iterate and substitute	for $\lvert x\rvert > 2$, $f(x) - x = (x - 2)(x + 1) > 0$ when $x > 2$ (and symmetrically for $x < -2$), so every solution lies in $[-2, 2]$; write $x = 2\cos\theta$, then $f(2\cos\theta) = 2\cos 2\theta$, so the n-fold iterate is $2\cos(2^3\theta)$: solving $\cos(2^3\theta) = \cos\theta$ on $[0, \pi]$ gives $\theta = \frac{2k\pi}{2^3 - 1}$ (4 values) or $\theta = \frac{2k\pi}{2^3 + 1}$ (5 values), overlapping only at $\theta = 0$: $8 = 2^3$ solutions
20	C	divisor-chain recurrence	$f(n) = \sum_{d \mid n, d < n} f(d)$ depends only on the multiset of exponents in n's prime factorisation; a direct count over $n = 1, \ldots, 150$ gives 7 values with $f(n) = 44$

Set 1, Paper 2

Q	Ans	Circuit	Reflex
1	H	inscribed angle	central angle is double; angles in the same segment are equal; opposite arc gives the supplement
2	B	converse / contrapositive	contrapositive: if $x^2 \leq 4$ then $x \leq 2$; it is true here
3	A	proof audit	the product can vanish because the cosine factor is 0; and line 3 also ignores other multiples of π
4	C	seasons	$\sin(2x)$ completes 2 full oscillations on $[0, 2\pi]$, two solutions each

Q	Ans	Circuit	Reflex
5	A	end behaviour	end behaviour: even degree has equal signs at $\pm\infty$, odd degree opposite; then compare with the sign of $p(0)$
6	D	AM-GM	AM-GM: $x+y \geq 2\sqrt{xy} = 12$, equality only at $x = y$
7	E	compartmentalise and negate	compartmentalise: flip every quantifier, then flip the final claim
8	B	larger set	draw the two sets; the sufficient condition is the smaller set
9	D	$\bigstar + \triangle = 2a$	add the two arguments: they must sum to 8 with equal values
10	G	statement testing	sketch the parabola from the given facts; look for a counterexample before believing a 'must'
11	G	midpoint chord	concave up: chords sit above the curve, tangents below, gradient increases
12	C	compartmentalise and negate	compartmentalise: flip every quantifier, then flip the final claim
13	D	touch or cross	odd multiplicity crosses, even multiplicity touches
14	E	$\bigstar + \triangle = 2a$	point symmetry about $(2, 0)$, the midpoint of the interval, kills the integral
15	B	shared axis, given root	$f(3) = f(-4)$ forces the axis of symmetry to $x = \frac{3+-4}{2} = -\frac{1}{2}$ regardless of a and c; reflecting the given root -5 in that axis gives the other root 4 (equal to it only if $r = -\frac{1}{2}$, which is excluded here)
16	B	compose two reflections	point symmetry about $(-2, 0)$ plus line symmetry about $x = 1$ compose: shifting by $6 = 2(1 - -2)$ sends $f(x) \mapsto 2b - f(x)$, and shifting by 12 returns $f(x)$ unchanged. From -3 to -21 is -3 such half-shifts, an odd count, so $f(-21) = 2b - f(-3) = -3$
17	B	Vieta on f'	sketch the sign of f' across each root: it changes sign at a simple root and holds its sign through a repeated one
18	D	range symmetry	$f(x) + f(2a - x) = 2b$ is point symmetry about $(3, 3)$, so the range is symmetric about 3: minimum $= 2b - M = 0$; the symmetry sends the maximiser $x = 0$ to $x = 2a - c = 6$: $0 + 6 = 6$
19	B	Vieta on f'	sketch the sign of f' across each root: it changes sign at a simple root and holds its sign through a repeated one
20	E	Vieta on f'	sketch the sign of f' across each root: it changes sign at a simple root and holds its sign through a repeated one