TMUA9.
MOCK EPSILON.

One set, Paper 1 and Paper 2, TMUA format.
Free. No NOIR PREMIERE required.
Official time 75 minutes. NOIR target: 15.

TMUA9.

How to use this mock

One set: a Paper 1 (Applications of Mathematical Knowledge) and a Paper 2 (Mathematical Reasoning), twenty multiple-choice questions each, in the TMUA format: pick one letter, no working marked, no calculator.

The point. The official paper gives you 75 minutes. A NOIR-trained student should be done in 15. Every question is one reaction circuit with nothing else in the way, named in cyan at the end of the question, except the last few in each paper, which chain two or three ideas together and are left unnamed on purpose: that is the part of the paper built to feel like a real Q15-20.

The rule. Set the timer for 15 minutes per paper. Stop when it rings, whatever is left is a miss. Mark from the key at the back: every answer comes with the circuit and the one-line reflex.

Verification. Every numeric answer was recomputed from the underlying integral, root, or coordinate geometry by a computer algebra system, independently of the shortcut; every logic answer was checked by brute force where a predicate could be evaluated.

Want more of this. This is one of eight free sets in the TMUA9 Question Bank. The full ten-set TMUA9. Papers, the DELUXE rehearsal, and NOIR PREMIERE go further: `https://tmua9.com/noir-premiere`.

Set 1 **Paper 1** Applications of Mathematical Knowledge

TMUA9. PAPERS

20 questions. Official time 75 minutes. NOIR target: 15 minutes. Every question is one reaction circuit, named in the margin. Answer on the grid at the end of the set.

1 The tangent to the curve $y = x^3 + bx$ (where b is a constant) at the point where $x = 2$ passes through the point $(5, 34)$, which does not lie on the curve. What is the x-intercept of this tangent? [ext. tangent]

A $\frac{8}{15}$ **B** $\frac{6}{5}$ **C** $\frac{8}{5}$
D $\frac{48}{25}$ **E** 2 **F** 5

2 The function $f(x) = \cos(\frac{\pi x}{p})$ is considered only on the closed interval $[19, 29]$. For natural numbers p and q with $19 \le q \le 29$, f attains its maximum value 1 at $x = q$. What is the least possible value of $p + q$? [$p+q$ search]

A 19 **B** 20 **C** 21 **D** 22 **E** 23 **F** 29

3 The roots of $x^2 - 8x + c = 0$ differ by 2. What is c? [Vieta]

A -15 **B** 13 **C** 15 **D** 16 **E** 20 **F** 225

4 Two sides of a triangle have lengths 15 and 9 and the angle between them is 120°. What is the length of the third side? [$120°$]

A $3\sqrt{19}$ **B** $3\sqrt{34}$ **C** 21 **D** 22 **E** 24 **F** 42

5 Tangents to $y = -2x^2 + bx + c$ at the points where $x = -2024$ and $x = -2018$ meet at the point R. What is the x-coordinate of R? [$S{:}T$]

A -4042 **B** -2024 **C** -2022 **D** -2021 **E** -2020 **F** -2018

6 A straight line meets the curve $y = 3x^3 - 6x^2 + cx + d$ at the points where $x = 1$, $x = 3$ and $x = k$. What is k? [root sum]

A -10 **B** -6 **C** -2 **D** -1 **E** 4 **F** 6

7 A cubic curve has a local maximum at $x = 2048$ and a local minimum at $x = 2054$. The tangent at the local maximum meets the curve again at the point where $x =$ [five points]

A 2045 **B** 2051 **C** 2054 **D** 2057 **E** $2054 + 3\sqrt{3}$ **F** 2060

8 What is the sum of all solutions of the equation $\cos 2x = \frac{5}{2}\cos x - \frac{7}{4}$ in the interval $0 \le x \le 2\pi$? [root-pair sum]

A $\frac{\pi}{3}$ **B** π **C** 2π **D** 3π **E** 4π **F** 24π

9 For how many integer values of c does the equation $2x^3 + 15x^2 + 24x + c = 0$ have three distinct real roots? [root band]

A 0 **B** 25 **C** 26 **D** 27 **E** 28 **F** 29

10 A sequence $a_1, a_2, a_3, \ldots$ satisfies $a_1 = 3$ and $a_n + a_{n+1} = -3n + 1$ for every positive integer n. What is a_9? [pair the recurrence]

A -27 **B** -14 **C** -12 **D** -10 **E** -9 **F** 3

11 A cubic curve $y = 3x^3 + ax^2 + bx$ (where a and b are constants) has a local maximum at $x = -1$ and a local minimum at $x = 0$. What is b? [Vieta on f']

A -2 **B** -1 **C** 0 **D** 1 **E** 2

12 What is the sum of all solutions of the equation $\cos 2x = -\cos x$ in the interval $0 \le x \le 2\pi$? [root-pair sum]

A $\frac{\pi}{4}$ **B** π **C** 2π **D** 3π **E** $1 + 3\pi$ **F** 4π

13 The curve $y = (x+3)^3 + 3$ and the lines $x = 0$ and $y = 3$ enclose a region. The rectangle with vertices $(-3, 3)$, $(0, 30)$, $(0, 30)$ and $(-3, 30)$ is divided by the curve into two parts. What is the ratio of the larger part to the smaller? [n:1]

A 2 : 1 **B** 3 : 1 **C** 4 : 1 **D** 5 : 1 **E** 6 : 1

14 A cubic curve $y = 2x^3 + ax^2 + bx$ (where a and b are constants) has a local maximum at $x = -3$ and a local minimum at $x = -2$. What is $a + b$? [Vieta on f']

A -51 **B** 17 **C** 50 **D** 51 **E** 52 **F** 2601

15 Functions f and g are defined for all real x by $f(x) = \frac{1}{2}x|x|$ and

$$g(x) = \begin{cases} a(x+3) & x < -3 \\ 0 & -3 \le x \le 3 \\ a(x-3) & x > 3 \end{cases}$$

where $a > 0$ is a constant. Let $h(x) = \int_0^x (g(u) - f(u))\, du$. The largest value of a for which h attains a local maximum or a local minimum value at exactly one value of x is k. When $a = k$, what is $k + h(4)$? [piecewise extremum]

A $-\frac{23}{3}$ **B** $-\frac{20}{9}$ **C** $-\frac{5}{3}$
D 6 **E** 9 **F** $\frac{41}{3}$

16 Two circles, of radii 4 and 1, touch one another and both touch a common straight line ℓ, on the same side of ℓ. Circles $C_1, C_2, C_3, \ldots$ are then inscribed in the successive gaps: each C_n touches ℓ, touches the circle of radius 4, and touches C_{n-1} (with C_0 the circle of radius 1). For how many values of n does C_n have radius greater than $\frac{1}{25}$? [circle chain]

A 6 **B** 7 **C** 8 **D** 14 **E** 28 **F** 49

17 Triangle ABC has $BC = a$, $CA = b$, $AB = c = 8$, and $a \cos A = b \cos B$. Given also that $a + b = 9$, what is the largest possible area of the triangle? [two-branch cosine rule]

A $\sqrt{17}$ **B** $\frac{17}{4}$ **C** $2\sqrt{17}$
D $\frac{17+8\sqrt{17}}{4}$ **E** $8\sqrt{17}$ **F** 36

18 Three distinct integers $a < b < c$ satisfy

$$a + b + c = -10, \qquad a^2 + b^2 + c^2 = 102.$$

What is the smallest of the three? [mean-shift Diophantine]

A -30 **B** -10 **C** -9 **D** -1 **E** 1 **F** 100

19 $f(x) = x^2 - 2$. How many real solutions does $\underbrace{f(f(\cdots f}_{3}(x)\cdots)) = x$ have? [iterate + substitution]

A 4 **B** 6 **C** 7 **D** 8 **E** 9 **F** 64

20 For each integer $n \ge 1$ let $f(n)$ be the number of lists of *distinct* positive integers which begin with 1, end with n, and in which every term except the last divides the term after it. (For example $f(1) = 1$, $f(2) = 1$ and $f(6) = 3$, the three lists being 1, 6 and 1, 2, 6 and 1, 3, 6.) How many integers n with $1 < n \le 80$ satisfy $f(n) = 2$? [divisor chain]

A 2 **B** 3 **C** 4 **D** 5 **E** 16 **F** 24

Set 1 **Paper 2** Mathematical Reasoning

TMUA9. PAPERS

20 questions. Official time 75 minutes. NOIR target: 15 minutes. Every question is one reaction circuit, named in the margin. Answer on the grid at the end of the set.

1 Which of the following is the negation of the statement "For all real x, if $x > 4$ then $x^2 > 16$."? [negate]

A There is a real x with $x \leq 4$ and $x^2 > 16$.
B For all real x, if $x > 4$ then $x^2 \leq 16$.
C There is a real x with $x > 4$ and $x^2 \leq 16$.
D For all real x, if $x^2 \leq 16$ then $x \leq 4$.
E There is a real x with $x > 4$ or $x^2 \leq 16$.
F For all real x, if $x \leq 4$ then $x^2 \leq 16$.

2 Consider the statements P: "$x = 2$" and Q: "$x^2 - 5x + 6 = 0$". Which of the following is true? [larger set]

A P is necessary but not sufficient for Q
B P is sufficient but not necessary for Q
C P is necessary and sufficient for Q
D P is neither necessary nor sufficient for Q

3 Let $f(x) = ax^2 + bx + c$ with $a > 0$ and minimum point $(2, 2)$. Which of the following must be true?
I $f(0) = 2$
II $f(x) \geq 2$ for all real x
III $f(3) = f(1)$ [statements]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

4 $p(x)$ is a polynomial of degree 4 with positive leading coefficient, and $p(0)$ is positive. Which of the following statements must be true?
I $p(x) = 0$ has at least two real roots
II $p(x) = 0$ has an even number of real roots (counted with multiplicity)
III $p(x) \to +\infty$ as $x \to -\infty$ [ends]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

5 Which of the following is a counterexample to the statement "If n is a positive integer then $n^2 + n + 7$ is prime"? [counterexample]

A $n = 1$ **B** $n = 3$ **C** $n = 24$ **D** $n = 26$ **E** $n = 38$

6 $a_1, a_2, a_3, \ldots$ is an arithmetic progression with non-zero common difference and S_n is the sum of its first n terms. Which of the following must be true?

I $a_4 a_8 = a_6^2$
II $a_8 - a_4 = 4(a_2 - a_1)$
III $a_4 + a_8 = 2a_6$ [2×middle]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

7 Which of the following is a sufficient condition for $\int_0^4 f(x)\,\mathrm{d}x = 0$? [★ + △]

A $f(2+x) = f(2-x)$ for all x
B $f(2+x) = -f(2-x)$ for all x
C $f(0) = -f(4)$
D $f(0) = f(4) = 0$
E $f(2) = 0$
F $f(-x) = -f(x)$ for all x

8 For real numbers x, which of the following describes the condition $x(x-1) > 0$ in relation to the condition $x > 1$? [larger set]

A necessary but not sufficient
B sufficient but not necessary
C necessary and sufficient
D neither necessary nor sufficient

9 Which of the following statements is/are true?

I $\log_3(3+3) = \log_3 3 + \log_3 3$
II $x > 2$ is a sufficient condition for $x^2 > 4$
III $\int_{-2}^{2} (x^3 + 1x)\,\mathrm{d}x = 0$ [statements]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

10 The equation $4x^3 - 18x^2 = c$ has three distinct real solutions. Which of the following is the complete set of possible values of c? [root band]

A $c < -54$ or $c > 0$ **B** $-54 < c < 0$ **C** $c < -54$
D $0 < c < 54$ **E** $c > 0$ **F** $-54 \le c \le 0$

11 Which of the following statements is/are true?

I $|a+b| \leq |a|+|b|$ for all real a, b
II The equation $\sin x = \frac{3}{2}$ has real solutions
III The graph of $y = (x-1)^6(x+2)$ touches the x-axis at $x = 1$ [statements]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

12 Which of the following is a counterexample to the statement "If n is a positive integer then n^2+n+3 is prime"? [counterexample]

A $n=1$ **B** $n=2$ **C** $n=4$ **D** $n=10$ **E** $n=22$

13 A function f satisfies $f(x)+f(-6-x)=4$ for every real x. Given that $f(3)=-6$, what is $f(-9)$? [range symmetry]

A -6 **B** -2 **C** 6 **D** 9 **E** 10 **F** 11

14 x and y are positive real numbers with $xy=16$. Which of the following must be true?

I $\dfrac{1}{x}+\dfrac{1}{y} \geq \frac{1}{2}$
II $x^2+y^2 \geq 32$
III $x+y=8$ [AM-GM]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

15 $f(x)=ax^2+bx+c$ where $a \neq 0$, and $f(-6)=f(-3)$. Given also that $f(-6)=0$, which of the following must be true?

I $f(-3)=0$
II $f(0)=0$
III the sum of the roots of $f(x)=0$ is -9 [axis + root]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

16 A function f is defined for all real x and satisfies $f(x)+f(-2-x)=10$ and $f(2+x)=f(2-x)$ for every real x. Given that $f(1)=3$, what is $f(19)$? [reflect twice]

A 3 **B** 7 **C** 9 **D** 10 **E** 28 **F** 84

17 A function f satisfies $f'(x) = 3x^3 + \frac{3x^2}{2} - 3x - \frac{3}{2}$ for all real x. Which of the following must be true?

I f has a local minimum at $x = -1$
II the equation $f'(x) = 0$ has a root strictly between -1 and 1
III f has a local maximum at $x = -1$ [Vieta on f']

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

18 A function f, continuous on the whole real line, satisfies $f(x) + f(-x) = 2$ for every real x. The maximum value of f is 9, attained only at $x = 1$. What is (the minimum value of f) + (the value of x at which that minimum occurs)? [range symmetry]

A -16 **B** -8 **C** -7 **D** -1 **E** 8 **F** 10

19 $f(x) = ax^2 + bx + c$ where $a \neq 0$, and $f(6) = f(-3)$. Given also that $f(6) = 0$, which of the following must be true?

I the equation $f(x) = 0$ has exactly two distinct real roots
II $f(6) = f(-3) = 0$
III $f(-3) = 0$ [axis + root]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

20 A function f is defined for all real x and satisfies $f(x) + f(6 - x) = 2$ and $f(x) = f(-x)$ for every real x. Given that $f(-1) = 4$, what is $f(-19)$? [reflect twice]

A -6 **B** -4 **C** -2 **D** 0 **E** 2 **F** 4

Answer key

Each answer with the circuit that fires and the one-line reflex. Mark, then for every miss write the circuit name next to the question and redo it the NOIR way.

Set 1, Paper 1

Q	Ans	Circuit	Reflex
1	C	external tangent	the tangent at $x = 2$ has gradient $3t^2 + b$ and y-intercept $-2t^3 = -16$; passing through $(5, 34)$ forces gradient 10, so $b = -2$ and the x-intercept is $\dfrac{2t^3}{3t^2 + b} = \frac{8}{5}$
2	C	interval-max search	$f = 1$ needs x an even multiple of p; the smallest even multiple of p landing in $[19, 29]$, minimised over $p = 1, 2, \ldots$, occurs at $(p, q) = (1, 20)$: $p + q = 21$
3	C	Vieta	$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$: $c = 15$
4	C	$120°$ / $60°$ triangles	$120°$: $c^2 = a^2 + b^2 + ab$, the 3-5-7 family
5	D	$S : T = 2 : 1$	x_R is the midpoint: -2021
6	C	root sum	root sum is $-b/a = 2$: $k = 2 - (1) - (3) = -2$
7	D	five key points	five key points, gap $d = 3$: max, inflection, min, crossing at $m + 3d = 2057$
8	E	root-pair sum	with $u = \cos x$: $2u^2 - (\frac{5}{2})u - (-\frac{3}{4}) = 0$ has roots $u = \frac{3}{4}, \frac{1}{2}$; a root with $-1 < u \leq 1$ contributes a pair of x-solutions summing to 2π (for $u = 1$ that pair is $x = 0$ and $x = 2\pi$, the trap most people miss), but $u = -1$ contributes only the single solution $x = \pi$: total 4π
9	C	root band	root band: c strictly between $-f(\max)$ and $-f(\min)$, $-16 < c < 11$: 26 integers
10	E	paired recurrence	unwind $a_{n+1} = (-3n + 1) - a_n$ from $a_1 = 3$ down to $a_9 = -9$
11	C	Vieta on f'	$f'(x) = 9x^2 + 9x$ must have roots $-1, 0$: Vieta on the derivative gives $a = \frac{9}{2}$, $b = 0$
12	D	root-pair sum	with $u = \cos x$: $2u^2 - (-1)u - (1) = 0$ has roots $u = \frac{1}{2}, -1$; a root with $-1 < u \leq 1$ contributes a pair of x-solutions summing to 2π (for $u = 1$ that pair is $x = 0$ and $x = 2\pi$, the trap most people miss), but $u = -1$ contributes only the single solution $x = \pi$: total 3π
13	B	$n : 1$ box	corner at the stationary or inflection point: $n : 1$ with $n = 3$
14	D	Vieta on f'	$f'(x) = 6x^2 + 30x + 36$ must have roots $-3, -2$: Vieta on the derivative gives $a = 15, b = 36$
15	C	piecewise extremum (parameter search)	$h' = g - f$ is odd and always flips sign at $x = 0$ (that extremum never depends on a). On $x \geq 3$, $h' = a(x - 3) - \frac{1}{2}x^2$ is a downward parabola peaking at $\frac{a^2}{4c} - aw$: the peak first touches 0 when $a = 4cw = k = 6$, and past that two more sign changes appear on each side by symmetry. $h(4) = -\frac{23}{3}$, so $k + h(4) = -\frac{5}{3}$
16	B	circle-chain counting	if circles of radii p, q touch a line on the same side and touch each other, $\frac{1}{\sqrt{w}} = \frac{1}{\sqrt{p}} + \frac{1}{\sqrt{q}}$ for a circle w wedged between them and the line; here $\frac{1}{\sqrt{r_n}} = \frac{1}{\sqrt{1}} + \frac{n}{\sqrt{4}}$, an arithmetic sequence, so $r_n > \frac{1}{25}$ for $n < 8 \approx 8.00$: 7 values
17	C	two-branch cosine rule	clear denominators in $a \cos A = b \cos B$: $a^2(b^2 + c^2 - a^2) = b^2(a^2 + c^2 - b^2)$ factors as $(a - b)(a + b)(a^2 + b^2 - c^2) = 0$, so $a = b$ or $C = 90°$ (never both, since $a + b > 0$). $a = b$ gives area $2\sqrt{17}$; $C = 90°$ with $a + b = 9$ gives $ab = \frac{17}{2}$, area $\frac{17}{4}$: the larger is $2\sqrt{17}$
18	B	mean-shift Diophantine search	shift to the mean $m = S/3$: with $x = 3a - S$ etc., $x + y + z = 0$ and $x^2 + y^2 + z^2 = 9Q - 3S^2 = 618$, i.e. $x^2 + xy + y^2 = 309$ after eliminating $z = -(x + y)$; a short integer search on this ellipse gives a single triple up to order: $(a, b, c) = (-10, -1, 1)$
19	D	iterate and substitute	for $\lvert x \rvert > 2$, $f(x) - x = (x - 2)(x + 1) > 0$ when $x > 2$ (and symmetrically for $x < -2$), so every solution lies in $[-2, 2]$; write $x = 2\cos\theta$, then $f(2\cos\theta) = 2\cos 2\theta$, so the n-fold iterate is $2\cos(2^3\theta)$: solving $\cos(2^3\theta) = \cos\theta$ on $[0, \pi]$ gives $\theta = \frac{2k\pi}{2^3 - 1}$ (4 values) or $\theta = \frac{2k\pi}{2^3 + 1}$ (5 values), overlapping only at $\theta = 0$: $8 = 2^3$ solutions
20	C	divisor-chain recurrence	$f(n) = \sum_{d \mid n, d < n} f(d)$ depends only on the multiset of exponents in n's prime factorisation; a direct count over $n = 1, \ldots, 80$ gives 4 values with $f(n) = 2$

Set 1, Paper 2

Q	Ans	Circuit	Reflex
1	C	compartmentalise and negate	compartmentalise: flip every quantifier, then flip the final claim
2	B	larger set	draw the two sets; the sufficient condition is the smaller set
3	G	statement testing	sketch the parabola from the given facts; look for a counterexample before believing a 'must'
4	G	end behaviour	end behaviour: even degree has equal signs at $\pm\infty$, odd degree opposite; then compare with the sign of $p(0)$
5	A	counterexample	$n = 1$ gives 9, which is not prime
6	G	double the middle	equidistant terms have the same sum; $S_n = n\times$ middle term
7	B	$\bigstar + \triangle = 2a$	point symmetry about $(2, 0)$, the midpoint of the interval, kills the integral
8	A	larger set	$x(x-1) > 0$ means $x < 0$ or $x > 1$: the larger set
9	G	statement testing	test each statement on its own; a single counterexample kills a 'for all'
10	B	root band	stationary values 0 and -54; the horizontal line $y = c$ must lie strictly between them
11	F	statement testing	test each statement on its own; a single counterexample kills a 'for all'
12	B	counterexample	$n = 2$ gives 9, which is not prime
13	E	range symmetry	substitute $x = 3$: $f(3) + f(-9) = 4$, so $f(-9) = 4 - (-6) = 10$
14	E	AM-GM	AM-GM: $x + y \geq 2\sqrt{xy} = 8$, equality only at $x = y$
15	F	shared axis, given root	$f(-6) = f(-3)$ forces the axis of symmetry to $x = \frac{-6+-3}{2} = -\frac{9}{2}$ regardless of a and c; reflecting the given root -6 in that axis gives the other root -3 (equal to it only if $r = -\frac{9}{2}$, which is excluded here)
16	B	compose two reflections	point symmetry about $(-1, 5)$ plus line symmetry about $x = 2$ compose: shifting by $6 = 2(2 - -1)$ sends $f(x) \mapsto 2b - f(x)$, and shifting by 12 returns $f(x)$ unchanged. From 1 to 19 is 3 such half-shifts, an odd count, so $f(19) = 2b - f(1) = 7$
17	E	Vieta on f'	sketch the sign of f' across each root: it changes sign at a simple root and holds its sign through a repeated one
18	B	range symmetry	$f(x) + f(2a - x) = 2b$ is point symmetry about $(0, 1)$, so the range is symmetric about 1: minimum $= 2b - M = -7$; the symmetry sends the maximiser $x = 1$ to $x = 2a - c = -1$: $-7 + -1 = -8$
19	H	shared axis, given root	$f(6) = f(-3)$ forces the axis of symmetry to $x = \frac{6+-3}{2} = \frac{3}{2}$ regardless of a and c; reflecting the given root 6 in that axis gives the other root -3 (equal to it only if $r = \frac{3}{2}$, which is excluded here)
20	C	compose two reflections	point symmetry about $(3, 1)$ plus line symmetry about $x = 0$ compose: shifting by $-6 = 2(0 - 3)$ sends $f(x) \mapsto 2b - f(x)$, and shifting by -12 returns $f(x)$ unchanged. From -1 to -19 is 3 such half-shifts, an odd count, so $f(-19) = 2b - f(-1) = -2$