TMUA9.
MOCK ETA.

One set, Paper 1 and Paper 2, TMUA format.

Free. No NOIR PREMIERE required.
Official time 75 minutes. NOIR target: 15.

TMUA9.

How to use this mock

One set: a Paper 1 (Applications of Mathematical Knowledge) and a Paper 2 (Mathematical Reasoning), twenty multiple-choice questions each, in the TMUA format: pick one letter, no working marked, no calculator.

The point. The official paper gives you 75 minutes. A NOIR-trained student should be done in 15. Every question is one reaction circuit with nothing else in the way, named in cyan at the end of the question, except the last few in each paper, which chain two or three ideas together and are left unnamed on purpose: that is the part of the paper built to feel like a real Q15-20.

The rule. Set the timer for 15 minutes per paper. Stop when it rings, whatever is left is a miss. Mark from the key at the back: every answer comes with the circuit and the one-line reflex.

Verification. Every numeric answer was recomputed from the underlying integral, root, or coordinate geometry by a computer algebra system, independently of the shortcut; every logic answer was checked by brute force where a predicate could be evaluated.

Want more of this. This is one of eight free sets in the TMUA9 Question Bank. The full ten-set TMUA9. Papers, the DELUXE rehearsal, and NOIR PREMIERE go further: `https://tmua9.com/noir-premiere`.

Set 1 **Paper 1** Applications of Mathematical Knowledge

TMUA9. PAPERS

20 questions. Official time 75 minutes. NOIR target: 15 minutes. Every question is one reaction circuit, named in the margin. Answer on the grid at the end of the set.

1 What is the sum of all solutions of the equation $\cos 2x = \frac{1}{2}\cos x + \frac{1}{2}$ in the interval $0 \le x \le 2\pi$? [root-pair sum]

A π **B** 2π **C** 3π **D** 4π **E** $1+4\pi$ **F** 24π

2 What is the value of $\sum_{k=0}^{100} \cos(5+90k)^\circ$? [seasons]

A $\sin 5^\circ + \cos 5^\circ$ **B** $-\cos 5^\circ$ **C** $\sin 5^\circ - \cos 5^\circ$
D $\cos 5^\circ$ **E** $2\sin 5^\circ$ **F** $\sin 5^\circ$

3 Tangents to $y = x^2 + bx + c$ at the points where $x = 4096$ and $x = 4098$ meet at the point R. What is the x-coordinate of R? [$S:T$]

A 4096 **B** $\frac{12290}{3}$ **C** 4097
D $\frac{12292}{3}$ **E** 4098 **F** 8194

4 α and β are the roots of $x^2 - 4x + 3 = 0$. What is $\frac{1}{\alpha} + \frac{1}{\beta}$? [Vieta]

A $-\frac{4}{3}$ **B** $\frac{1}{3}$ **C** $\frac{3}{4}$
D $\frac{8}{9}$ **E** $\frac{4}{3}$ **F** 4

5 In triangle ABC, the cevians AD, BE and CF meet at a single point, with $\dfrac{BD}{DC} = 3$ and $\dfrac{CE}{EA} = \frac{1}{2}$. What is $\dfrac{AF}{FB}$? [Ceva]

A $\frac{1}{6}$ **B** $\frac{2}{7}$ **C** $\frac{2}{3}$
D $\frac{8}{9}$ **E** $\frac{3}{2}$ **F** 6

6 A cubic curve $y = -x^3 + ax^2 + bx$ (where a and b are constants) has a local maximum at $x = 1$ and a local minimum at $x = -2$. What is ab? [Vieta on f']

A -36 **B** -11 **C** -9 **D** -7 **E** -3 **F** 81

7 The curve $y = 2x^2 + bx + c$ meets the line $y = k$ at $x = -981$ and $x = -975$. What is the area of the region bounded by the curve, the line $y = k$ and the line $x = -984$? [1:2:1]

A 36 **B** 54 **C** 72 **D** 108 **E** 216 **F** 432

8 What is the coefficient of x^2y^2 in the expansion of $(2 + x + y)^4$? [multinomial]

A -6 **B** 4 **C** 6 **D** 12 **E** 24 **F** 36

9 The tangent to the curve $y = x^3 + bx$ (where b is a constant) at the point where $x = 1$ passes through the point $(5, 28)$, which does not lie on the curve. What is the x-intercept of this tangent? [ext. tangent]

A $-\frac{5}{3}$ **B** $\frac{5}{18}$ **C** $\frac{1}{3}$
D $\frac{2}{3}$ **E** 1 **F** 5

10 The curve $y = 12x^3 + bx^2 + cx + d$ meets the line $y = k$ at $x = 4096$, $x = 4100$ and $x = 4103$. What is the area of the region enclosed between the curve and the line for $4096 \le x \le 4100$? [lobes]

A 160 **B** 297 **C** 638 **D** 640 **E** 937 **F** 2560

11 A straight line is a tangent to $y = x^3 - 3x^2 + cx + d$ at the point where $x = -2$ and meets the curve again where $x = k$. What is k? [root sum]

A -7 **B** -1 **C** 1 **D** 5 **E** 7 **F** 42

12 An infinite sequence of similar shapes is drawn, each with sides $\frac{1}{2}$ of the previous one. The first has area 64. What is the total area of all the shapes? [shading]

A 48 **B** 64 **C** $\frac{256}{3}$ **D** 128 **E** $\frac{512}{3}$ **F** 256

13 What is $\int_{-2}^{2} (-x^3 + x^2 + 5x)\,dx$? [odd]

A 0 **B** $\frac{8}{3}$ **C** $\frac{13}{3}$
D $\frac{16}{3}$ **E** $\frac{26}{3}$ **F** $\frac{76}{3}$

14 The curve $y = \cos x$ is reflected in the line $y = 1$ and then translated by $\frac{\pi}{6}$ in the positive x-direction. What is the equation of the resulting curve? [vertex]

A $y = 2 - \cos(x - \frac{\pi}{6})$ **B** $y = 2 - \cos(x + \frac{\pi}{6})$
C $y = 1 - \cos(x - \frac{\pi}{6})$ **D** $y = 2 + \cos(x - \frac{\pi}{6})$
E $y = -1 - \cos(x - \frac{\pi}{6})$ **F** $y = 2 - \cos x - \frac{\pi}{6}$

15 Functions f and g are defined for all real x by $f(x) = 1x|x|$ and

$$g(x) = \begin{cases} a(x+2) & x < -2 \\ 0 & -2 \le x \le 2 \\ a(x-2) & x > 2 \end{cases}$$

where $a > 0$ is a constant. Let $h(x) = \int_0^x (g(u) - f(u))\,du$. The largest value of a for which h attains a local maximum or a local minimum value at exactly one value of x is k. When $a = k$, what is $k + h(4)$? [piecewise extremum]

A $-\frac{16}{3}$ **B** $\frac{8}{3}$ **C** 4 **D** 8 **E** 10 **F** $\frac{40}{3}$

16 Two circles, of radii 9 and 4, touch one another and both touch a common straight line ℓ, on the same side of ℓ. Circles $C_1, C_2, C_3, \ldots$ are then inscribed in the successive gaps: each C_n touches ℓ, touches the circle of radius 9, and touches C_{n-1} (with C_0 the circle of radius 4). For how many values of n does C_n have radius greater than $\frac{1}{100}$? [circle chain]

A 7 **B** 14 **C** 26 **D** 27 **E** 28 **F** 29

17 Triangle ABC has $BC = a$, $CA = b$, $AB = c = 10$, and $a\cos A = b\cos B$. Given also that $a + b = 14$, what is the largest possible area of the triangle? [two-branch cosine rule]

A $\frac{20\sqrt{6}}{3}$ **B** 24 **C** $10\sqrt{6}$
D $1 + 10\sqrt{6}$ **E** $24 + 10\sqrt{6}$ **F** 70

18 Three distinct integers $a < b < c$ satisfy

$$a + b + c = 16, \qquad a^2 + b^2 + c^2 = 394.$$

What is the largest of the three? [mean-shift Diophantine]

A -13 **B** -9 **C** 12 **D** 13 **E** 16 **F** 169

19 $f(x) = x^2 - 2$. How many real solutions does $\underbrace{f(f(\cdots f}_{3}(x)\cdots)) = x$ have? [iterate + substitution]

A 4 **B** 6 **C** 7 **D** 8 **E** 9 **F** 10

20 For each integer $n \geq 1$ let $f(n)$ be the number of lists of *distinct* positive integers which begin with 1, end with n, and in which every term except the last divides the term after it. (For example $f(1) = 1$, $f(2) = 1$ and $f(6) = 3$, the three lists being $1, 6$ and $1, 2, 6$ and $1, 3, 6$.) How many integers n with $1 < n \leq 100$ satisfy $f(n) = 32$?

[divisor chain]

A 1 **B** 2 **C** 3 **D** 4 **E** 6 **F** 12

Set 1 Paper 2 Mathematical Reasoning

TMUA9. PAPERS

20 questions. Official time 75 minutes. NOIR target: 15 minutes. Every question is one reaction circuit, named in the margin. Answer on the grid at the end of the set.

1 Which of the following is a counterexample to the statement "If $f'(c) = 0$ then f has a turning point at $x = c$"? [counterexample]

A $f(x) = x^2 - 2x,\ c = 1$
B $f(x) = x^2,\ c = 0$
C $f(x) = \sin x,\ c = \frac{\pi}{2}$
D $f(x) = x^3,\ c = 0$
E $f(x) = |x|,\ c = 0$

2 Which of the following is the contrapositive of the statement "if $x > 2$ then $x^2 > 4$"? [converse]

A if $x^2 \le 4$ then $x > 2$
B if $x^2 > 4$ then $x > 2$
C if $x^2 \le 4$ then $x \le 2$
D if $x^2 > 4$ then $x \le 2$
E if $x > 2$ then $x^2 > 4$
F if $x \le 2$ then $x^2 \le 4$

3 Consider the statements P: "$x^3 > 8$" and Q: "$x > 2$". Which of the following is true? [larger set]

A P is necessary but not sufficient for Q
B P is sufficient but not necessary for Q
C P is necessary and sufficient for Q
D P is neither necessary nor sufficient for Q

4 The function f is twice differentiable with $f''(x) > 0$ for all x in $[0, 1]$. Which of the following must be true?

I $\dfrac{f(0) + f(1)}{2} > f(\tfrac{1}{2})$
II the tangent to $y = f(x)$ at $x = \frac{1}{2}$ lies below the curve on $[0, 1]$
III $f(\frac{1}{2}) < f(0)$ [chord]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

5 Which of the following is the negation of the statement "For every real number x, $x^2 + 1 > 4x$."? [negate]

A For every real number x, $x^2 + 1 < 4x$.
B There is no real number x for which $x^2 + 1 > 4x$.
C There is a real number x for which $x^2 + 1 \le 4x$.
D There is a real number x for which $x^2 + 1 < 4x$.
E There is a real number x for which $x^2 + 1 > 4x$.
F For every real number x, $x^2 + 1 \le 4x$.

6 For the curve $y = (x-3)^2 x^4 (x-2)$, which of the following is true when x is a large negative number? [ends]

A y is positive and $|y|$ is large
B y is negative and $|y|$ is large
C y is close to 0
D y is positive and close to 1
E y alternates in sign

7 The equation $-2x^3 + 9x^2 - 12x = c$ has three distinct real solutions. Which of the following is the complete set of possible values of c? [root band]

A $4 < c < 5$ **B** $c < -5$ or $c > -4$ **C** $-5 < c < -4$
D $-5 \leq c \leq -4$ **E** $c < -5$ **F** $c > -4$

8 A function f satisfies $f(x) + f(-2-x) = -4$ for every real x. Given that $f(-3) = 2$, what is $f(1)$? [range symmetry]

A -36 **B** -24 **C** -6 **D** -2 **E** 2 **F** 6

9 x and y are positive real numbers with $x + y = 4$. Which of the following must be true?

I $\sqrt{xy} \geq 2$
II $x^2 + y^2 \geq 8$
III $xy \geq 4$ [AM-GM]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

10 Which of the following statements is/are true?

I Every quartic polynomial has at least one real root
II The graph of $y = (x-1)^6(x+2)$ touches the x-axis at $x = 1$
III The curve $y = x^3$ has two turning points [statements]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

11 $a_1, a_2, a_3, \ldots$ is an arithmetic progression with non-zero common difference and S_n is the sum of its first n terms. Which of the following must be true?

I $S_9 = 9a_5$
II $a_4 + a_6 = a_{10}$
III $a_6 - a_4 = 2(a_2 - a_1)$ [2×middle]

A none of them
B I only
C II only
D III only
E I and II only

F I and III only

G II and III only

H I, II and III

12 $p(x)$ is a polynomial of degree 3 with negative leading coefficient, and $p(0)$ is negative. Which of the following statements must be true?

I $p(x) = 0$ has at least two real roots
II $p(x) = 0$ has an even number of real roots (counted with multiplicity)
III $p(x) \to -\infty$ as $x \to -\infty$ [ends]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

13 For real numbers x, which of the following describes the condition $x > 0$ in relation to the condition $x > 3$? [larger set]

A necessary but not sufficient

B sufficient but not necessary

C necessary and sufficient

D neither necessary nor sufficient

14 Which of the following is a sufficient condition for $\int_0^6 f(x)\,dx = 0$? [$\star + \triangle$]

A $f(0) = -f(6)$

B $f(-x) = -f(x)$ for all x

C $f(3+x) = f(3-x)$ for all x

D $f(3) = 0$

E $f(3+x) = -f(3-x)$ for all x

F $f(0) = f(6) = 0$

15 $f(x) = ax^2 + bx + c$ where $a \neq 0$, and $f(1) = f(0)$. Given also that $f(1) = 0$, which of the following must be true?

I $f(\frac{1}{2})$ is the least value of f if $a > 0$, and the greatest if $a < 0$
II the equation $f(x) = 0$ has exactly two distinct real roots
III $f(0) = 0$ [axis + root]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

16 A function f is defined for all real x and satisfies $f(x) + f(4-x) = -2$ and $f(x) = f(-x)$ for every real x. Given that $f(0) = 0$, what is $f(-28)$? [reflect twice]

A -24 **B** -6 **C** -4 **D** -2 **E** 0 **F** 2

17 A function f satisfies $f'(x) = \frac{x^3}{2} - \frac{3x^2}{2} - \frac{x}{2} + \frac{3}{2}$ for all real x. Which of the following must be true?

I f has a local maximum at $x = -1$
II the curve $y = f(x)$ has exactly three turning points
III the equation $f'(x) = 0$ has a root strictly between -1 and 1 [Vieta on f']

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

18 A function f, continuous on the whole real line, satisfies $f(x) + f(-2-x) = 2$ for every real x. The maximum value of f is 8, attained only at $x = 0$. What is (the minimum value of f) + (the value of x at which that minimum occurs)? [range symmetry]

A -32 **B** -10 **C** -8 **D** -6 **E** -2 **F** 8

19 A function f is defined for all real x and satisfies $f(x) + f(-6-x) = -8$ and $f(3+x) = f(3-x)$ for every real x. Given that $f(1) = 2$, what is $f(-59)$? [reflect twice]

A -30 **B** -10 **C** -8 **D** 2 **E** 10 **F** 14

20 A function f is defined for all real x and satisfies $f(x) + f(4-x) = -4$ and $f(-1+x) = f(-1-x)$ for every real x. Given that $f(-1) = 4$, what is $f(-19)$? [reflect twice]

A -10 **B** -8 **C** -6 **D** -4 **E** -2 **F** 4

Answer key

Each answer with the circuit that fires and the one-line reflex. Mark, then for every miss write the circuit name next to the question and redo it the NOIR way.

Set 1, Paper 1

Q	Ans	Circuit	Reflex
1	D	root-pair sum	with $u=\cos x$: $2u^2-(\frac{1}{2})u-(\frac{3}{2})=0$ has roots $u=1,-\frac{3}{4}$; a root with $-1<u\leq 1$ contributes a pair of x-solutions summing to 2π (for $u=1$ that pair is $x=0$ and $x=2\pi$, the trap most people miss), but $u=-1$ contributes only the single solution $x=\pi$: total 4π
2	D	seasons	four consecutive terms cancel; 101 terms leave 1: $\cos 5°$
3	C	$S:T=2:1$	x_R is the midpoint: 4097
4	E	Vieta	$\frac{\alpha+\beta}{\alpha\beta}=\frac{4}{3}$
5	C	Ceva	Ceva: the product of the three ratios is 1
6	C	Vieta on f'	$f'(x)=-3x^2-3x+6$ must have roots $-2,1$: Vieta on the derivative gives $a=-\frac{3}{2}, b=6$
7	C	1:2:1	strip half a lobe wide equals the lobe: $\frac{\lvert a\rvert}{6}(6)^3=72$
8	C	multinomial	$\frac{4!}{2!\,2!\,0!}\cdot(2)^0=6$
9	C	external tangent	the tangent at $x=1$ has gradient $3t^2+b$ and y-intercept $-2t^3=-2$; passing through $(5,28)$ forces gradient 6, so $b=3$ and the x-intercept is $\dfrac{2t^3}{3t^2+b}=\frac{1}{3}$
10	D	unequal lobes	$S_\bigstar=\frac{\lvert a\rvert}{6}\bigstar^3(\frac{\bigstar}{2}+\triangle)$ with $\bigstar=4$, $\triangle=3$: 640
11	E	root sum	tangency counts twice: $k=3-2(-2)=7$
12	C	geometric shading	areas scale by r^2: $\frac{64}{1-(\frac{1}{2})^2}=\frac{256}{3}$
13	D	odd function, zero integral	odd powers vanish on $[-k,k]$: $2\times\frac{1k^3}{3}$
14	A	vertex tracking	reflect in $y=1$: $y\to 2-y$; then $x\to x-\frac{\pi}{6}$
15	B	piecewise extremum (parameter search)	$h'=g-f$ is odd and always flips sign at $x=0$ (that extremum never depends on a). On $x\geq 2$, $h'=a(x-2)-1x^2$ is a downward parabola peaking at $\frac{a^2}{4c}-aw$: the peak first touches 0 when $a=4cw=k=8$, and past that two more sign changes appear on each side by symmetry. $h(4)=-\frac{16}{3}$, so $k+h(4)=\frac{8}{3}$
16	E	circle-chain counting	if circles of radii p,q touch a line on the same side and touch each other, $\frac{1}{\sqrt{w}}=\frac{1}{\sqrt{p}}+\frac{1}{\sqrt{q}}$ for a circle w wedged between them and the line; here $\frac{1}{\sqrt{r_n}}=\frac{1}{\sqrt{4}}+\frac{n}{\sqrt{9}}$, an arithmetic sequence, so $r_n>\frac{1}{100}$ for $n<\frac{57}{2}\approx 28.50$: 28 values
17	C	two-branch cosine rule	clear denominators in $a\cos A=b\cos B$: $a^2(b^2+c^2-a^2)=b^2(a^2+c^2-b^2)$ factors as $(a-b)(a+b)(a^2+b^2-c^2)=0$, so $a=b$ or $C=90°$ (never both, since $a+b>0$). $a=b$ gives area $10\sqrt{6}$; $C=90°$ with $a+b=14$ gives $ab=48$, area 24: the larger is $10\sqrt{6}$
18	D	mean-shift Diophantine search	shift to the mean $m=S/3$: with $x=3a-S$ etc., $x+y+z=0$ and $x^2+y^2+z^2=9Q-3S^2=2778$, i.e. $x^2+xy+y^2=1389$ after eliminating $z=-(x+y)$; a short integer search on this ellipse gives a single triple up to order: $(a,b,c)=(-9,12,13)$
19	D	iterate and substitute	for $\lvert x\rvert>2$, $f(x)-x=(x-2)(x+1)>0$ when $x>2$ (and symmetrically for $x<-2$), so every solution lies in $[-2,2]$; write $x=2\cos\theta$, then $f(2\cos\theta)=2\cos 2\theta$, so the n-fold iterate is $2\cos(2^3\theta)$: solving $\cos(2^3\theta)=\cos\theta$ on $[0,\pi]$ gives $\theta=\frac{2k\pi}{2^3-1}$ (4 values) or $\theta=\frac{2k\pi}{2^3+1}$ (5 values), overlapping only at $\theta=0$: $8=2^3$ solutions
20	A	divisor-chain recurrence	$f(n)=\sum_{d\mid n,\,d<n}f(d)$ depends only on the multiset of exponents in n's prime factorisation; a direct count over $n=1,\ldots,100$ gives 1 values with $f(n)=32$

Set 1, Paper 2

Q	Ans	Circuit	Reflex
1	D	counterexample	x^3 has a stationary inflection at 0
2	C	converse / contrapositive	contrapositive: if $x^2\leq 4$ then $x\leq 2$; it is true here
3	C	larger set	draw the two sets; the sufficient condition is the smaller set

Q	Ans	Circuit	Reflex
4	E	midpoint chord	concave up: chords sit above the curve, tangents below, gradient increases
5	C	compartmentalise and negate	compartmentalise: flip every quantifier, then flip the final claim
6	B	end behaviour	degree 7 with positive leading coefficient: sign at $-\infty$ is $(-1)^7$
7	C	root band	stationary values -4 and -5; the horizontal line $y = c$ must lie strictly between them
8	C	range symmetry	substitute $x = -3$: $f(-3) + f(1) = -4$, so $f(1) = -4 - (2) = -6$
9	C	AM-GM	AM-GM: $\sqrt{xy} \leq \frac{x+y}{2} = 2$; $x^2 + y^2 \geq \frac{(x+y)^2}{2}$
10	C	statement testing	test each statement on its own; a single counterexample kills a 'for all'
11	F	double the middle	equidistant terms have the same sum; $S_n = n\times$ middle term
12	A	end behaviour	end behaviour: even degree has equal signs at $\pm\infty$, odd degree opposite; then compare with the sign of $p(0)$
13	A	larger set	the larger set is the necessary condition: $x > 3$ is the smaller set
14	E	$\bigstar + \triangle = 2a$	point symmetry about $(3, 0)$, the midpoint of the interval, kills the integral
15	H	shared axis, given root	$f(1) = f(0)$ forces the axis of symmetry to $x = \frac{1+0}{2} = \frac{1}{2}$ regardless of a and c; reflecting the given root 1 in that axis gives the other root 0 (equal to it only if $r = \frac{1}{2}$, which is excluded here)
16	D	compose two reflections	point symmetry about $(2, -1)$ plus line symmetry about $x = 0$ compose: shifting by $-4 = 2(0-2)$ sends $f(x) \mapsto 2b - f(x)$, and shifting by -8 returns $f(x)$ unchanged. From 0 to -28 is 7 such half-shifts, an odd count, so $f(-28) = 2b - f(0) = -2$
17	C	Vieta on f'	sketch the sign of f' across each root: it changes sign at a simple root and holds its sign through a repeated one
18	C	range symmetry	$f(x) + f(2a - x) = 2b$ is point symmetry about $(-1, 1)$, so the range is symmetric about 1: minimum $= 2b - M = -6$; the symmetry sends the maximiser $x = 0$ to $x = 2a - c = -2$: $-6 + -2 = -8$
19	B	compose two reflections	point symmetry about $(-3, -4)$ plus line symmetry about $x = 3$ compose: shifting by $12 = 2(3 - -3)$ sends $f(x) \mapsto 2b - f(x)$, and shifting by 24 returns $f(x)$ unchanged. From 1 to -59 is -5 such half-shifts, an odd count, so $f(-59) = 2b - f(1) = -10$
20	B	compose two reflections	point symmetry about $(2, -2)$ plus line symmetry about $x = -1$ compose: shifting by $-6 = 2(-1-2)$ sends $f(x) \mapsto 2b - f(x)$, and shifting by -12 returns $f(x)$ unchanged. From -1 to -19 is 3 such half-shifts, an odd count, so $f(-19) = 2b - f(-1) = -8$