TMUA9.
MOCK GAMMA.

One set, Paper 1 and Paper 2, TMUA format.

Free. No NOIR PREMIERE required.
Official time 75 minutes. NOIR target: 15.

TMUA9.

How to use this mock

One set: a Paper 1 (Applications of Mathematical Knowledge) and a Paper 2 (Mathematical Reasoning), twenty multiple-choice questions each, in the TMUA format: pick one letter, no working marked, no calculator.

The point. The official paper gives you 75 minutes. A NOIR-trained student should be done in 15. Every question is one reaction circuit with nothing else in the way, named in cyan at the end of the question, except the last few in each paper, which chain two or three ideas together and are left unnamed on purpose: that is the part of the paper built to feel like a real Q15-20.

The rule. Set the timer for 15 minutes per paper. Stop when it rings, whatever is left is a miss. Mark from the key at the back: every answer comes with the circuit and the one-line reflex.

Verification. Every numeric answer was recomputed from the underlying integral, root, or coordinate geometry by a computer algebra system, independently of the shortcut; every logic answer was checked by brute force where a predicate could be evaluated.

Want more of this. This is one of eight free sets in the TMUA9 Question Bank. The full ten-set TMUA9. Papers, the DELUXE rehearsal, and NOIR PREMIERE go further: `https://tmua9.com/noir-premiere`.

TMUA9.

Set 1

Paper 1 Applications of Mathematical Knowledge

TMUA9. PAPERS

20 questions. Official time 75 minutes. NOIR target: 15 minutes. Every question is one reaction circuit, named in the margin. Answer on the grid at the end of the set.

1 A geometric progression has sum to infinity $\frac{9}{2}$, and the sum to infinity of the squares of its terms is $\frac{81}{8}$. What is the first term? [middle²]

A 1 **B** $\frac{9}{4}$ **C** 3 **D** $\frac{9\sqrt{2}}{4}$ **E** 4 **F** 18

2 The tangent to the curve $y = x^3 + bx$ (where b is a constant) at the point where $x = 1$ passes through the point $(-1, -13)$, which does not lie on the curve. What is the x-intercept of this tangent? [ext. tangent]

A -1 **B** $\frac{1}{66}$ **C** $\frac{1}{22}$
D $\frac{2}{11}$ **E** 1 **F** $\frac{12}{11}$

3 What is the sum of all solutions of the equation $\cos 2x = -\frac{8}{3}\cos x - \frac{5}{3}$ in the interval $0 \le x \le 2\pi$? [root-pair sum]

A $\frac{3\pi}{4}$ **B** π **C** 2π **D** $3\pi - 1$ **E** 3π **F** 4π

4 A cubic curve has a local maximum at $x = -981$ and a local minimum at $x = -975$. The tangent at the local maximum meets the curve again at the point where $x =$ [five points]

A -984 **B** -978 **C** -975 **D** -972 **E** $3\sqrt{3} - 975$ **F** -969

5 The curve $y = \frac{1}{2}x(x-10)(x+10)$ encloses a region with the x-axis between $x = 0$ and $x = 10$. The region between the curve and the x-axis from $x = 10$ to $x = k$ (with $k > 10$) has the same area. What is k? [$\int = 0$]

A 11 **B** $10\sqrt{2}$ **C** 15 **D** $1 + 10\sqrt{2}$ **E** $10\sqrt{3}$ **F** 20

6 A cubic curve $y = -2x^3 + ax^2 + bx$ (where a and b are constants) has a local maximum at $x = -1$ and a local minimum at $x = -2$. What is b? [Vieta on f']

A -144 **B** -24 **C** -18 **D** -12 **E** -9 **F** -8

7 What is the area of the finite region enclosed between the curve $y = -2x^2 + 2x - 4$ and the line $y = -12x + 8$? [$\sqrt{D}/|a|$]

A $\frac{125}{36}$ **B** $\frac{125}{18}$ **C** $\frac{125}{3}$
D $\frac{128}{3}$ **E** $\frac{500}{3}$ **F** $\frac{15625}{9}$

8 Two circles have radii 2 and 1 and their centres are 13 apart. A common tangent touches both circles on the same side of the line of centres. What is the distance between the two points of contact? [tangent]

A $4\sqrt{10}$ **B** $\sqrt{165}$ **C** $2\sqrt{42}$
D 13 **E** $\sqrt{170}$ **F** $4\sqrt{42}$

9 The tangent to the curve $y = x^3 + bx$ (where b is a constant) at the point where $x = -3$ passes through the point $(-1, 31)$, which does not lie on the curve. What is the x-intercept of this tangent? [ext. tangent]

A $-\frac{648}{23}$ **B** $-\frac{162}{23}$ **C** -3
D $-\frac{54}{23}$ **E** $-\frac{31}{23}$ **F** -1

10 A sequence $a_1, a_2, a_3, \ldots$ satisfies $a_1 = 1$ and $a_n + a_{n+1} = -4n + 3$ for every positive integer n. What is a_7? [pair the recurrence]

A -132 **B** -15 **C** -14 **D** -12 **E** -11 **F** -10

11 The function $f(x) = \cos\left(\frac{\pi x}{p}\right)$ is considered only on the closed interval $[17, 29]$. For natural numbers p and q with $17 \leq q \leq 29$, f attains its maximum value 1 at $x = q$. What is the least possible value of $p + q$? [$p + q$ search]

A 17 **B** 18 **C** 19 **D** 20 **E** 21 **F** 29

12 α and β are the roots of $x^2 + x - 6 = 0$. What is $\frac{1}{\alpha} + \frac{1}{\beta}$? [Vieta]

A $-\frac{11}{6}$ **B** -1 **C** $-\frac{1}{6}$
D $\frac{1}{36}$ **E** $\frac{1}{6}$ **F** 6

13 What is the sum of all solutions of the equation $\cos 2x = \frac{3}{2}\cos x - \frac{5}{4}$ in the interval $0 \leq x \leq 2\pi$? [root-pair sum]

A π **B** 2π **C** 3π **D** $4\pi - 1$ **E** 4π **F** 48π

14 How many distinct real roots does the equation $4x^3 - 6x^2 + (4) = 0$ have? [root band]

A 0 **B** 1 **C** 2 **D** 3

15 Functions f and g are defined for all real x by $f(x) = 1x|x|$ and

$$g(x) = \begin{cases} a(x+2) & x < -2 \\ 0 & -2 \leq x \leq 2 \\ a(x-2) & x > 2 \end{cases}$$

where $a > 0$ is a constant. Let $h(x) = \int_0^x (g(u) - f(u))\,du$. The largest value of a for which h attains a local maximum or a local minimum value at exactly one value of x is k. When $a = k$, what is $k + h(3)$? [piecewise extremum]

A -5 **B** 3 **C** 8 **D** 10 **E** 13 **F** 36

16 Two circles, of radii 9 and 4, touch one another and both touch a common straight line ℓ, on the same side of ℓ. Circles $C_1, C_2, C_3, \ldots$ are then inscribed in the successive gaps: each C_n touches ℓ, touches the circle of radius 9, and touches C_{n-1} (with C_0 the circle of radius 4). For how many values of n does C_n have radius greater than $\frac{1}{50}$? [circle chain]

A 18 **B** 19 **C** 20 **D** 21 **E** 38 **F** 114

17 Triangle ABC has $BC = a$, $CA = b$, $AB = c = 6$, and $a\cos A = b\cos B$. Given also that $a + b = 7$, what is the largest possible area of the triangle? [two-branch cosine rule]

A $\frac{13}{4}$ **B** $\frac{3\sqrt{13}-2}{2}$
C $\frac{3\sqrt{13}}{2}$ **D** $\frac{13+6\sqrt{13}}{4}$
E $\frac{9\sqrt{13}}{2}$ **F** 21

18 Three distinct integers $a < b < c$ satisfy

$$a + b + c = 8, \qquad a^2 + b^2 + c^2 = 416.$$

What is the largest of the three? [mean-shift Diophantine]

A -12 **B** 4 **C** 8 **D** 16 **E** 17 **F** 256

19 $f(x) = x^2 - 2$. How many real solutions does $\underbrace{f(f(\cdots f}_{4}(x)\cdots)) = x$ have? [iterate + substitution]

A 8 **B** 15 **C** 16 **D** 17 **E** 32 **F** 256

20 For each integer $n \geq 1$ let $f(n)$ be the number of lists of *distinct* positive integers which begin with 1, end with n, and in which every term except the last divides the term after it. (For example $f(1) = 1$, $f(2) = 1$ and $f(6) = 3$, the three lists being $1, 6$ and $1, 2, 6$ and $1, 3, 6$.) How many integers n with $1 < n \leq 200$ satisfy $f(n) = 13$? [divisor chain]

A 18 **B** 19 **C** 20 **D** 38 **E** 76 **F** 228

Set 1 Paper 2 Mathematical Reasoning

TMUA9. PAPERS

20 questions. Official time 75 minutes. NOIR target: 15 minutes. Every question is one reaction circuit, named in the margin. Answer on the grid at the end of the set.

1 Which of the following is the contrapositive of the statement "if $x > 5$ then $x^2 > 25$"? [converse]

A if $x^2 > 25$ then $x \leq 5$
B if $x^2 \leq 25$ then $x > 5$
C if $x > 5$ then $x^2 > 25$
D if $x^2 \leq 25$ then $x \leq 5$
E if $x^2 > 25$ then $x > 5$
F if $x \leq 5$ then $x^2 \leq 25$

2 A, B and P lie on a circle with centre O, and $\angle APB = 40°$. Which of the following must be true?

I $\angle AQB = 140°$ for every point Q on the other arc
II $\angle AOB = 80°$
III $\angle AQB = 40°$ for every point Q on the same arc as P [inscribed]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

3 $p(x)$ is a polynomial of degree 3 with positive leading coefficient, and $p(0)$ is positive. Which of the following statements must be true?

I $p(x) = 0$ has an even number of real roots (counted with multiplicity)
II $p(x) = 0$ has at least two real roots
III $p(x) = 0$ has at least one real root [ends]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

4 Claim: if $a > b$ then $\frac{1}{a} < \frac{1}{b}$. Consider the following argument.

1 Suppose $a > b$
2 Divide both sides by ab, which preserves the inequality
3 so $\frac{1}{b} > \frac{1}{a}$
4 hence $\frac{1}{a} < \frac{1}{b}$

Which of the following best describes the argument? [proof]

A The argument is incorrect: the first error is in line 2
B The argument is incorrect: the first error is in line 1
C The argument is incorrect: the first error is in line 3

D The argument is incorrect: the first error is in line 4

E The argument is correct

5 x and y are positive real numbers with $x + y = 8$. Which of the following must be true?

I $xy \leq 16$

II $\frac{1}{x} + \frac{1}{y} \geq \frac{1}{2}$

III $x^2 + y^2 \geq 32$ [AM-GM]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

6 The function f is twice differentiable with $f''(x) > 0$ for all x in $[1, 5]$. Which of the following must be true?

I $f(3) < f(1)$

II the trapezium rule with one strip overestimates $\int_1^5 f(x)\,dx$

III the tangent to $y = f(x)$ at $x = 3$ lies below the curve on $[1, 5]$ [chord]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

7 Which of the following is a counterexample to the statement "If n is a multiple of 4 and of 6 then n is a multiple of 24"? [counterexample]

A $n = 12$ **B** $n = 27$ **C** $n = 53$ **D** $n = 59$ **E** $n = 71$

8 Which of the following is the negation of the statement "For all real x, if $x > 3$ then $x^2 > 9$."? [negate]

A There is a real x with $x > 3$ or $x^2 \leq 9$.

B For all real x, if $x > 3$ then $x^2 \leq 9$.

C For all real x, if $x^2 \leq 9$ then $x \leq 3$.

D There is a real x with $x > 3$ and $x^2 \leq 9$.

E There is a real x with $x \leq 3$ and $x^2 > 9$.

F For all real x, if $x \leq 3$ then $x^2 \leq 9$.

9 Which of the following statements is/are true?

I A geometric series with common ratio $-\frac{1}{3}$ has a sum to infinity

II $\int_{-2}^{2} (x^3 + 5x)\,dx = 0$

III $x^2 + 4x + 5 > 0$ for all real x [statements]

A none of them

B I only

C II only

D III only

E I and II only
F I and III only
G II and III only
H I, II and III

10 $a_1, a_2, a_3, \ldots$ is an arithmetic progression with non-zero common difference and S_n is the sum of its first n terms. Which of the following must be true?

I $a_3 + a_{16} = a_8 + a_{11}$
II $S_{10} = 2S_5$
III $S_9 = 9a_5$ [2×middle]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

11 A function f satisfies $f(x) + f(4 - x) = -2$ for every real x. Given that $f(-1) = -3$, what is $f(5)$? [range symmetry]

A -5 **B** -3 **C** -1 **D** 1 **E** 3 **F** 12

12 $b_1, b_2, b_3, \ldots$ is a geometric progression with common ratio r ($r \neq 0, r \neq \pm 1$) and S_n is the sum of its first n terms. Which of the following must be true?

I $b_8 = b_4\, r^4$
II $b_4\, b_8 = b_6^2$
III if $|r| < 1$ then the sum to infinity is $\dfrac{b_1}{1 - r}$ [middle²]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

13 A function f satisfies $f(x) + f(-2 - x) = 2$ for every real x. Given that $f(4) = 4$, what is $f(-6)$? [range symmetry]

A -24 **B** -4 **C** -2 **D** -1 **E** 4 **F** 6

14 Let $f(x) = ax^2 + bx + c$ with $a > 0$ and minimum point $(1, 3)$. Which of the following must be true?

I $b^2 < 4ac$
II $f(x) > 0$ for all real x
III $f(x) \geq 3$ for all real x [statements]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

15 $f(x) = ax^2 + bx + c$ where $a \neq 0$, and $f(6) = f(5)$. Given also that $f(-2) = 0$, which of the following must be true?

I $f(\frac{11}{2})$ is the least value of f if $a > 0$, and the greatest if $a < 0$
II $f(13) = 0$
III $f(0) = 0$

[axis + root]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

16 A function f is defined for all real x and satisfies $f(x) + f(-2-x) = 10$ and $f(2+x) = f(2-x)$ for every real x. Given that $f(0) = -3$, what is $f(-30)$?

[reflect twice]

A -3 **B** 3 **C** 10 **D** 13 **E** 15 **F** 26

17 A function f satisfies $f'(x) = 3x^3 + 9x^2 - 3x - 9$ for all real x. Which of the following must be true?

I f has a local minimum at $x = 1$
II f has a local minimum at $x = -1$
III the equation $f'(x) = 0$ has a root strictly between -1 and 1

[Vieta on f']

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

18 A function f, continuous on the whole real line, satisfies $f(x) + f(-x) = 6$ for every real x. The maximum value of f is 6, attained only at $x = 2$. What is (the minimum value of f) + (the value of x at which that minimum occurs)?

[range symmetry]

A -4 **B** -2 **C** 0 **D** 2 **E** 4 **F** 8

19 A function f is defined for all real x and satisfies $f(x) + f(-6-x) = -4$ and $f(-1+x) = f(-1-x)$ for every real x. Given that $f(2) = -2$, what is $f(22)$?

[reflect twice]

A -6 **B** -4 **C** -2 **D** 0 **E** 2 **F** 4

20 $f(x) = ax^2 + bx + c$ where $a \neq 0$, and $f(1) = f(3)$. Given also that $f(-2) = 0$, which of the following must be true?

I $f(2)$ is the least value of f if $a > 0$, and the greatest if $a < 0$
II the equation $f(x) = 0$ has exactly two distinct real roots
III the sum of the roots of $f(x) = 0$ is 4

[axis + root]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

Answer key

Each answer with the circuit that fires and the one-line reflex. Mark, then for every miss write the circuit name next to the question and redo it the NOIR way.

Set 1, Paper 1

Q	Ans	Circuit	Reflex
1	C	square the middle	$\frac{S_\infty^2}{S_\infty^{(2)}} = \frac{1+r}{1-r}$ gives $r = \frac{1}{3}$, then $a = 3$
2	D	external tangent	the tangent at $x = 1$ has gradient $3t^2 + b$ and y-intercept $-2t^3 = -2$; passing through $(-1, -13)$ forces gradient 11, so $b = 8$ and the x-intercept is $\dfrac{2t^3}{3t^2+b} = \frac{2}{11}$
3	E	root-pair sum	with $u = \cos x$: $2u^2 - (-\frac{8}{3})u - (-\frac{2}{3}) = 0$ has roots $u = -1, -\frac{1}{3}$; a root with $-1 < u \le 1$ contributes a pair of x-solutions summing to 2π (for $u = 1$ that pair is $x = 0$ and $x = 2\pi$, the trap most people miss), but $u = -1$ contributes only the single solution $x = \pi$: total 3π
4	D	five key points	five key points, gap $d = 3$: max, inflection, min, crossing at $m + 3d = -972$
5	B	integral zero ($k = \sqrt{2}m$)	equal areas means $\int_0^k = 0$: $k^2 = 2m^2$, $k = \sqrt{2}\,m$
6	D	Vieta on f'	$f'(x) = -6x^2 - 18x - 12$ must have roots $-2, -1$: Vieta on the derivative gives $a = -9$, $b = -12$
7	C	$\bigstar = \sqrt{D}/\lvert a\rvert$	$D = 100$, $\bigstar = \sqrt{D}/\lvert a\rvert = 5$, $S = \frac{\lvert a\rvert}{6}\bigstar^3 = \frac{125}{3}$
8	C	common tangent	external tangent: $\sqrt{d^2 - (R-r)^2}$
9	D	external tangent	the tangent at $x = -3$ has gradient $3t^2 + b$ and y-intercept $-2t^3 = 54$; passing through $(-1, 31)$ forces gradient 23, so $b = -4$ and the x-intercept is $\dfrac{2t^3}{3t^2+b} = -\frac{54}{23}$
10	E	paired recurrence	unwind $a_{n+1} = (-4n+3) - a_n$ from $a_1 = 1$ down to $a_7 = -11$
11	C	interval-max search	$f = 1$ needs x an even multiple of p; the smallest even multiple of p landing in $[17, 29]$, minimised over $p = 1, 2, \ldots$, occurs at $(p, q) = (1, 18)$: $p + q = 19$
12	E	Vieta	$\frac{\alpha+\beta}{\alpha\beta} = \frac{-1}{-6}$
13	E	root-pair sum	with $u = \cos x$: $2u^2 - (\frac{3}{2})u - (-\frac{1}{4}) = 0$ has roots $u = \frac{1}{4}, \frac{1}{2}$; a root with $-1 < u \le 1$ contributes a pair of x-solutions summing to 2π (for $u = 1$ that pair is $x = 0$ and $x = 2\pi$, the trap most people miss), but $u = -1$ contributes only the single solution $x = \pi$: total 4π
14	B	root band	stationary values 4 and 2; count the sign pattern: 1
15	B	piecewise extremum (parameter search)	$h' = g - f$ is odd and always flips sign at $x = 0$ (that extremum never depends on a). On $x \ge 2$, $h' = a(x-2) - 1x^2$ is a downward parabola peaking at $\frac{a^2}{4c} - aw$: the peak first touches 0 when $a = 4cw = k = 8$, and past that two more sign changes appear on each side by symmetry. $h(3) = -5$, so $k + h(3) = 3$
16	B	circle-chain counting	if circles of radii p, q touch a line on the same side and touch each other, $\frac{1}{\sqrt{w}} = \frac{1}{\sqrt{p}} + \frac{1}{\sqrt{q}}$ for a circle w wedged between them and the line; here $\frac{1}{\sqrt{r_n}} = \frac{1}{\sqrt{4}} + \frac{n}{\sqrt{9}}$, an arithmetic sequence, so $r_n > \frac{1}{50}$ for $n < \frac{30\sqrt{2}-3}{2} \approx 19.71$: 19 values
17	C	two-branch cosine rule	clear denominators in $a\cos A = b\cos B$: $a^2(b^2 + c^2 - a^2) = b^2(a^2 + c^2 - b^2)$ factors as $(a-b)(a+b)(a^2 + b^2 - c^2) = 0$, so $a = b$ or $C = 90^\circ$ (never both, since $a + b > 0$). $a = b$ gives area $\frac{3\sqrt{13}}{2}$; $C = 90^\circ$ with $a + b = 7$ gives $ab = \frac{13}{2}$, area $\frac{13}{4}$: the larger is $\frac{3\sqrt{13}}{2}$
18	D	mean-shift Diophantine search	shift to the mean $m = S/3$: with $x = 3a - S$ etc., $x + y + z = 0$ and $x^2 + y^2 + z^2 = 9Q - 3S^2 = 3552$, i.e. $x^2 + xy + y^2 = 1776$ after eliminating $z = -(x+y)$; a short integer search on this ellipse gives a single triple up to order: $(a, b, c) = (-12, 4, 16)$
19	C	iterate and substitute	for $\lvert x\rvert > 2$, $f(x) - x = (x-2)(x+1) > 0$ when $x > 2$ (and symmetrically for $x < -2$), so every solution lies in $[-2, 2]$; write $x = 2\cos\theta$, then $f(2\cos\theta) = 2\cos 2\theta$, so the n-fold iterate is $2\cos(2^4\theta)$: solving $\cos(2^4\theta) = \cos\theta$ on $[0, \pi]$ gives $\theta = \frac{2k\pi}{2^4-1}$ (8 values) or $\theta = \frac{2k\pi}{2^4+1}$ (9 values), overlapping only at $\theta = 0$: $16 = 2^4$ solutions
20	B	divisor-chain recurrence	$f(n) = \sum_{d\mid n, d<n} f(d)$ depends only on the multiset of exponents in n's prime factorisation; a direct count over $n = 1, \ldots, 200$ gives 19 values with $f(n) = 13$

Set 1, Paper 2

Q	Ans	Circuit	Reflex
1	D	converse / contrapositive	contrapositive: if $x^2 \leq 25$ then $x \leq 5$; it is true here
2	H	inscribed angle	central angle is double; angles in the same segment are equal; opposite arc gives the supplement
3	D	end behaviour	end behaviour: even degree has equal signs at $\pm\infty$, odd degree opposite; then compare with the sign of $p(0)$
4	A	proof audit	dividing by ab reverses the inequality when $ab < 0$; $a = 1, b = -1$ is a counterexample to the claim
5	H	AM-GM	AM-GM: $\sqrt{xy} \leq \frac{x+y}{2} = 4$; $x^2 + y^2 \geq \frac{(x+y)^2}{2}$
6	G	midpoint chord	concave up: chords sit above the curve, tangents below, gradient increases
7	A	counterexample	$n = 12$ is a multiple of both but not of 24: the lowest common multiple is 12
8	D	compartmentalise and negate	compartmentalise: flip every quantifier, then flip the final claim
9	H	statement testing	test each statement on its own; a single counterexample kills a 'for all'
10	F	double the middle	equidistant terms have the same sum; $S_n = n\times$ middle term
11	D	range symmetry	substitute $x = -1$: $f(-1) + f(5) = -2$, so $f(5) = -2 - (-3) = 1$
12	H	square the middle	square the middle; blocks of a GP are earlier blocks times r^k
13	C	range symmetry	substitute $x = 4$: $f(4) + f(-6) = 2$, so $f(-6) = 2 - (4) = -2$
14	H	statement testing	sketch the parabola from the given facts; look for a counterexample before believing a 'must'
15	E	shared axis, given root	$f(6) = f(5)$ forces the axis of symmetry to $x = \frac{6+5}{2} = \frac{11}{2}$ regardless of a and c; reflecting the given root -2 in that axis gives the other root 13 (equal to it only if $r = \frac{11}{2}$, which is excluded here)
16	D	compose two reflections	point symmetry about $(-1, 5)$ plus line symmetry about $x = 2$ compose: shifting by $6 = 2(2 - -1)$ sends $f(x) \mapsto 2b - f(x)$, and shifting by 12 returns $f(x)$ unchanged. From 0 to -30 is -5 such half-shifts, an odd count, so $f(-30) = 2b - f(0) = 13$
17	B	Vieta on f'	sketch the sign of f' across each root: it changes sign at a simple root and holds its sign through a repeated one
18	B	range symmetry	$f(x) + f(2a - x) = 2b$ is point symmetry about $(0, 3)$, so the range is symmetric about 3: minimum $= 2b - M = 0$; the symmetry sends the maximiser $x = 2$ to $x = 2a - c = -2$: $0 + -2 = -2$
19	C	compose two reflections	point symmetry about $(-3, -2)$ plus line symmetry about $x = -1$ compose: shifting by $4 = 2(-1 - -3)$ sends $f(x) \mapsto 2b - f(x)$, and shifting by 8 returns $f(x)$ unchanged. From 2 to 22 is 5 such half-shifts, an odd count, so $f(22) = 2b - f(2) = -2$
20	H	shared axis, given root	$f(1) = f(3)$ forces the axis of symmetry to $x = \frac{1+3}{2} = 2$ regardless of a and c; reflecting the given root -2 in that axis gives the other root 6 (equal to it only if $r = 2$, which is excluded here)