TMUA9.
MOCK THETA.

One set, Paper 1 and Paper 2, TMUA format.

Free. No NOIR PREMIERE required.
Official time 75 minutes. NOIR target: 15.

TMUA9.

How to use this mock

One set: a Paper 1 (Applications of Mathematical Knowledge) and a Paper 2 (Mathematical Reasoning), twenty multiple-choice questions each, in the TMUA format: pick one letter, no working marked, no calculator.

The point. The official paper gives you 75 minutes. A NOIR-trained student should be done in 15. Every question is one reaction circuit with nothing else in the way, named in cyan at the end of the question, except the last few in each paper, which chain two or three ideas together and are left unnamed on purpose: that is the part of the paper built to feel like a real Q15-20.

The rule. Set the timer for 15 minutes per paper. Stop when it rings, whatever is left is a miss. Mark from the key at the back: every answer comes with the circuit and the one-line reflex.

Verification. Every numeric answer was recomputed from the underlying integral, root, or coordinate geometry by a computer algebra system, independently of the shortcut; every logic answer was checked by brute force where a predicate could be evaluated.

Want more of this. This is one of eight free sets in the TMUA9 Question Bank. The full ten-set TMUA9. Papers, the DELUXE rehearsal, and NOIR PREMIERE go further: `https://tmua9.com/noir-premiere`.

Set 1 Paper 1 Applications of Mathematical Knowledge

TMUA9. PAPERS

20 questions. Official time 75 minutes. NOIR target: 15 minutes. Every question is one reaction circuit, named in the margin. Answer on the grid at the end of the set.

1 The function $f(x) = \cos(\frac{\pi x}{p})$ is considered only on the closed interval $[9, 15]$. For natural numbers p and q with $9 \le q \le 15$, f attains its maximum value 1 at $x = q$. What is the least possible value of $p + q$? [$p+q$ search]

A 9 **B** 10 **C** 11 **D** 12 **E** 13 **F** 15

2 A cubic curve $y = ax^3 + bx^2 + cx + d$ touches the line $y = k$ at its local minimum, where $x = -4092$, and crosses $y = k$ where $x = -4098$. At what value of x is its local maximum? [five points]

A -4098 **B** -4096 **C** -4095 **D** -4094 **E** -4090 **F** 4096

3 α and β are the roots of $x^2 - 2x - 3 = 0$. What is $\frac{1}{\alpha} + \frac{1}{\beta}$? [Vieta]

A $-\frac{3}{2}$ **B** $-\frac{2}{3}$ **C** $-\frac{2}{9}$
D $\frac{4}{9}$ **E** $\frac{2}{3}$ **F** 2

4 The curve $y = x^3 + bx^2 + cx + d$ meets the line $y = k$ at $x = 1961$, $x = 1965$ and $x = 1970$. What is the area of the region enclosed between the curve and the line for $1961 \le x \le 1965$? [lobes]

A $\frac{112}{9}$ **B** $\frac{448}{9}$ **C** $\frac{224}{3}$
D $\frac{1625}{12}$ **E** $\frac{2521}{12}$ **F** 448

5 The curve $y = 2x^2 + bx + c$ meets the line $y = k$ at $x = -981$ and $x = -977$. The region enclosed by the curve, the line $y = k$ and the vertical line $x = m$ (where $m > -977$) has the same area as the region enclosed by the curve and the line $y = k$. What is m? [1:2:1]

A -1300 **B** -976 **C** $2\sqrt{3} - 979$
D -975 **E** $2\sqrt{3} - 977$ **F** -973

6 For how many integer values of c does the equation $4x^3 - 54x^2 + 216x + c = 0$ have three distinct real roots? [root band]

A 0 **B** 52 **C** 53 **D** 54 **E** 55 **F** 56

7 The graph of $y = g(x)$ is obtained from $y = f(x)$ by a reflection in the y-axis followed by a translation of 2 units in the negative x-direction. The graph of $y = h(x)$ is obtained by the same two transformations in the opposite order. Which condition is necessary and sufficient for $g(x) = h(x)$ for all x? [period]

A $f(x) = f(x+2)$ **B** $f(x) = f(x+4)$ **C** $f(x) = f(x+8)$
D $f(x) = f(-x)$ **E** $f(x) = f(4-x)$ **F** $f(x) = f(2-x)$

8 A sequence $a_1, a_2, a_3, \ldots$ satisfies $a_1 = 3$ and $a_n + a_{n+1} = -2n$ for every positive integer n. What is a_9? [pair the recurrence]

A -15 **B** -11 **C** -7 **D** -6 **E** -5 **F** -3

9 In an arithmetic sequence, $a_2 = 6$ and $a_{10} = 8$. What is a_6? [grasshopper]

A 1 **B** 6 **C** 7 **D** 8 **E** 9 **F** 14

10 In a geometric sequence, $b_2 = 18$ and $b_5 = 144$. What is the common ratio? [middle^2]

A -2 **B** 2 **C** $\frac{8}{3}$ **D** $2\sqrt{2}$ **E** 8 **F** 24

11 The curve $y = -2x^2 + bx + c$ and the line $y = -3x + k$ intersect where $x = -1024$ and $x = -1021$. What is the area of the finite region enclosed between them? [★^{3}]

A 3 **B** 9 **C** 10 **D** 18 **E** 54 **F** 108

12 The tangent to the curve $y = x^3 + bx$ (where b is a constant) at the point where $x = -3$ passes through the point $(1, 80)$, which does not lie on the curve. What is the x-intercept of this tangent? [ext. tangent]

A $-\frac{54}{13}$ **B** -3 **C** $-\frac{27}{13}$
D $-\frac{45}{26}$ **E** $-\frac{81}{52}$ **F** 1

13 What is the value of $\sum_{k=0}^{2024} \cos(10 + 90k)^\circ$? [seasons]

A $\sin 10^\circ - \cos 10^\circ$ **B** $\sin 10^\circ$
C $-\cos 10^\circ$ **D** $2 \sin 10^\circ$
E $\sin 10^\circ + \cos 10^\circ$ **F** $\cos 10^\circ$

14 A sequence $a_1, a_2, a_3, \ldots$ satisfies $a_1 = -1$ and $a_n + a_{n+1} = -4n + 3$ for every positive integer n. What is a_6? [pair the recurrence]

A -12 **B** -11 **C** -9 **D** -8 **E** -7 **F** 8

15 Functions f and g are defined for all real x by $f(x) = 1x|x|$ and

$$g(x) = \begin{cases} a(x+2) & x < -2 \\ 0 & -2 \le x \le 2 \\ a(x-2) & x > 2 \end{cases}$$

where $a > 0$ is a constant. Let $h(x) = \int_0^x \big(g(u) - f(u)\big)\, du$. The largest value of a for which h attains a local maximum or a local minimum value at exactly one value of x is k. When $a = k$, what is $k + h(4)$? [piecewise extremum]

A $-\frac{16}{3}$ **B** $\frac{2}{3}$ **C** $\frac{8}{3}$
D 8 **E** 10 **F** $\frac{40}{3}$

16 Two circles, of radii 4 and 1, touch one another and both touch a common straight line ℓ, on the same side of ℓ. Circles $C_1, C_2, C_3, \ldots$ are then inscribed in the successive gaps: each C_n touches ℓ, touches the circle of radius 4, and touches C_{n-1} (with C_0 the circle of radius 1). For how many values of n does C_n have radius greater than $\frac{1}{25}$? [circle chain]

A 5 **B** 6 **C** 7 **D** 8 **E** 21 **F** 49

17 Triangle ABC has $BC = a$, $CA = b$, $AB = c = 10$, and $a \cos A = b \cos B$. Given also that $a + b = 14$, what is the largest possible area of the triangle? [two-branch cosine rule]

A $\frac{10\sqrt{6}}{3}$ **B** $5\sqrt{6}$ **C** 24
D $10\sqrt{6}$ **E** $24 + 10\sqrt{6}$ **F** 70

18 Three distinct integers $a < b < c$ satisfy

$$a + b + c = -2, \qquad a^2 + b^2 + c^2 = 292.$$

What is the middle of the three? [mean-shift Diophantine]

A -12 **B** -8 **C** -4 **D** -2 **E** -1 **F** 12

19 $f(x) = x^2 - 2$. How many real solutions does $\underbrace{f(f(\cdots f}_{3}(x)\cdots)) = x$ have? [iterate + substitution]

A 2 **B** 4 **C** 6 **D** 7 **E** 8 **F** 9

20 For each integer $n \ge 1$ let $f(n)$ be the number of lists of *distinct* positive integers which begin with 1, end with n, and in which every term except the last divides the term after it. (For example $f(1) = 1$, $f(2) = 1$ and $f(6) = 3$, the three lists being 1, 6 and 1, 2, 6 and 1, 3, 6.) How many integers n with $1 < n \le 100$ satisfy $f(n) = 2$? [divisor chain]

A 2 **B** 3 **C** 4 **D** 5 **E** 8 **F** 48

Set 1 Paper 2 Mathematical Reasoning

TMUA9. PAPERS

20 questions. Official time 75 minutes. NOIR target: 15 minutes. Every question is one reaction circuit, named in the margin. Answer on the grid at the end of the set.

1 A function f satisfies $f(x) + f(-2-x) = -8$ for every real x. Given that $f(5) = -3$, what is $f(-7)$? [range symmetry]

A -30 **B** -11 **C** -6 **D** -5 **E** -3 **F** 3

2 Which of the following is a counterexample to the statement "If $f''(c) = 0$ then $(c, f(c))$ is a point of inflection"? [counterexample]

A $f(x) = x^4$, $c = 0$
B $f(x) = x^3$, $c = 0$
C $f(x) = x^2$, $c = 0$
D $f(x) = \sin x$, $c = 0$
E $f(x) = x^3 - 3x$, $c = 0$

3 Which of the following statements is/are true?

I $\int_{-3}^{3} (x^2 + 2)\,dx = 0$
II Every quartic polynomial has at least one real root
III $x^2 > 2$ is a sufficient condition for $x > \sqrt{2}$ [statements]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

4 $p(x)$ is a polynomial of degree 3 with positive leading coefficient, and $p(0)$ is positive. Which of the following statements must be true?

I $p(x) = 0$ has at least two real roots
II $p(x) = 0$ has an even number of real roots (counted with multiplicity)
III $p(x) = 0$ has at least one real root [ends]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

5 A, B and P lie on a circle with centre O, and $\angle APB = 70°$. Which of the following must be true?

I $\angle AQB = 110°$ for every point Q on the other arc
II $\angle AOB = 140°$
III OA is perpendicular to AB

[inscribed]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

6 x and y are positive real numbers with $x + y = 6$. Which of the following must be true?

I $\frac{1}{x} + \frac{1}{y} \geq \frac{2}{3}$
II $\sqrt{xy} \geq 3$
III $xy \leq 9$

[AM-GM]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

7 $b_1, b_2, b_3, \ldots$ is a geometric progression with common ratio r ($r \neq 0, r \neq \pm 1$) and S_n is the sum of its first n terms. Which of the following must be true?

I $b_7 + b_8 = 2b_7$
II if $|r| < 1$ then the sum to infinity is $\dfrac{b_1}{1-r}$
III $b_7\, b_9 = b_8\, b_8$

[middle^2]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

8 For real numbers x, which of the following describes the condition $x^2 > 4$ in relation to the condition $x > 2$?

[larger set]

A necessary but not sufficient
B sufficient but not necessary
C necessary and sufficient
D neither necessary nor sufficient

9 Which of the following is the negation of the statement "There is an integer n such that $n^2 < 2n$."? [negate]

A There is an integer n such that $n^2 > 2n$.

B There is no integer n such that $n^2 \geq 2n$.

C For every integer n, $n^2 > 2n$.

D For every integer n, $n^2 \geq 2n$.

E For every integer n, $n^2 < 2n$.

F There is an integer n such that $n^2 \geq 2n$.

10 Which of the following conditions is/are sufficient for the graph of $y = f(x)$ to have the line $x = 3$ as an axis of symmetry?

I $f(3 - x) = f(x)$ for all x
II $f(6 - x) = f(x)$ for all x
III $f(3 + x) + f(3 - x) = 0$ for all x [★ + △]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

11 How many times does the graph of $y = (x + 1)(x + 3)(x - 4)$ cross the x-axis? [touch/cross]

A 0 **B** 1 **C** 2 **D** 3 **E** 4

12 Which of the following statements is/are true?

I $\int_{-3}^{3} (x^3 + 5x)\,\mathrm{d}x = 0$
II $\sqrt{a^2} = a$ for all real a
III A geometric series with common ratio -1 has a sum to infinity [statements]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

13 Which of the following is a sufficient condition for $\int_{1}^{5} f(x)\,\mathrm{d}x = 0$? [★ + △]

A $f(-x) = -f(x)$ for all x

B $f(3 + x) = -f(3 - x)$ for all x

C $f(3 + x) = f(3 - x)$ for all x

D $f(1) = -f(5)$

E $f(1) = f(5) = 0$

F $f(3) = 0$

14 Claim: for all real x, $x^3 \geq x$. Consider the following argument.

1 $x^3 - x = x(x-1)(x+1)$
2 for $x \geq 1$ all three factors are non-negative
3 so $x^3 - x \geq 0$ for $x \geq 1$
4 hence $x^3 \geq x$ for all real x

Which of the following best describes the argument? [proof]

A The argument is incorrect: the first error is in line 4

B The argument is incorrect: the first error is in line 1

C The argument is incorrect: the first error is in line 2

D The argument is incorrect: the first error is in line 3

E The argument is correct

15 $f(x) = ax^2 + bx + c$ where $a \neq 0$, and $f(6) = f(4)$. Given also that $f(4) = 0$, which of the following must be true?

I $f(6) = 0$
II the sum of the roots of $f(x) = 0$ is 10
III the equation $f(x) = 0$ has exactly two distinct real roots [axis + root]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

16 A function f is defined for all real x and satisfies $f(x) + f(-6-x) = -8$ and $f(x) = f(-x)$ for every real x. Given that $f(0) = 1$, what is $f(-30)$? [reflect twice]

A -27 **B** -9 **C** -8 **D** -6 **E** 1 **F** 7

17 A function f satisfies $f'(x) = \frac{x^3}{2} - x^2 - \frac{x}{2} + 1$ for all real x. Which of the following must be true?

I f has a local minimum at $x = -1$
II f has a local minimum at $x = 1$
III f has a local maximum at $x = 1$ [Vieta on f']

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

18 A function f, continuous on the whole real line, satisfies $f(x) + f(-x) = 2$ for every real x. The maximum value of f is 6, attained only at $x = 3$. What is (the minimum value of f) + (the value of x at which that minimum occurs)? [range symmetry]

A -28 **B** -8 **C** -7 **D** -4 **E** -3 **F** 9

19 A function f, continuous on the whole real line, satisfies $f(x) + f(-x) = -4$ for every real x. The maximum value of f is 5, attained only at $x = -3$. What is (the minimum value of f) + (the value of x at which that minimum occurs)? [range symmetry]

A -9 **B** -8 **C** -6 **D** 2 **E** 3 **F** 36

20 $f(x) = ax^2 + bx + c$ where $a \neq 0$, and $f(-1) = f(-2)$. Given also that $f(-4) = 0$, which of the following must be true?

I $f(-1) = f(-2) = 0$
II the equation $f(x) = 0$ has exactly two distinct real roots
III $f(1) = 0$

[axis + root]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Answer key

Each answer with the circuit that fires and the one-line reflex. Mark, then for every miss write the circuit name next to the question and redo it the NOIR way.

Set 1, Paper 1

Q	Ans	Circuit	Reflex
1	C	interval-max search	$f = 1$ needs x an even multiple of p; the smallest even multiple of p landing in $[9, 15]$, minimised over $p = 1, 2, \ldots$, occurs at $(p, q) = (1, 10)$: $p + q = 11$
2	B	five key points	tangent span is 3 gaps, the other extremum sits 2 gaps from the crossing: $x = -4096$
3	B	Vieta	$\frac{\alpha+\beta}{\alpha\beta} = \frac{2}{-3}$
4	C	unequal lobes	$S_{\bigstar} = \frac{\lvert a\rvert}{6}\bigstar^3(\frac{\bigstar}{2} + \triangle)$ with $\bigstar = 4$, $\triangle = 5$: $\frac{224}{3}$
5	D	1:2:1	1:2:1, the outer strip is half a lobe wide: $m = -977 + 2 = -975$
6	C	root band	root band: c strictly between $-f(\max)$ and $-f(\min)$, $-270 < c < -216$: 53 integers
7	B	reflect-translate period	$g(x) = f(-x-2)$, $h(x) = f(-x+2)$: equal iff f has period 4
8	E	paired recurrence	unwind $a_{n+1} = (-2n) - a_n$ from $a_1 = 3$ down to $a_9 = -5$
9	C	grasshopper	the middle term is the mean of equidistant terms: $\frac{6+(8)}{2}$
10	B	square the middle	$b_5/b_2 = r^3 = 8$
11	B	$\bigstar^3$ chord area	a line is a line: $\bigstar = 3$, $S = \frac{\lvert a\rvert}{6}\bigstar^3 = 9$
12	C	external tangent	the tangent at $x = -3$ has gradient $3t^2 + b$ and y-intercept $-2t^3 = 54$; passing through $(1, 80)$ forces gradient 26, so $b = -1$ and the x-intercept is $\dfrac{2t^3}{3t^2 + b} = -\frac{27}{13}$
13	F	seasons	four consecutive terms cancel; 2025 terms leave 1: $\cos 10°$
14	D	paired recurrence	unwind $a_{n+1} = (-4n + 3) - a_n$ from $a_1 = -1$ down to $a_6 = -8$
15	C	piecewise extremum (parameter search)	$h' = g - f$ is odd and always flips sign at $x = 0$ (that extremum never depends on a). On $x \geq 2$, $h' = a(x-2) - 1x^2$ is a downward parabola peaking at $\frac{a^2}{4c} - aw$: the peak first touches 0 when $a = 4cw = k = 8$, and past that two more sign changes appear on each side by symmetry. $h(4) = -\frac{16}{3}$, so $k + h(4) = \frac{8}{3}$
16	C	circle-chain counting	if circles of radii p, q touch a line on the same side and touch each other, $\frac{1}{\sqrt{w}} = \frac{1}{\sqrt{p}} + \frac{1}{\sqrt{q}}$ for a circle w wedged between them and the line; here $\frac{1}{\sqrt{r_n}} = \frac{1}{\sqrt{1}} + \frac{n}{\sqrt{4}}$, an arithmetic sequence, so $r_n > \frac{1}{25}$ for $n < 8 \approx 8.00$: 7 values
17	D	two-branch cosine rule	clear denominators in $a\cos A = b\cos B$: $a^2(b^2 + c^2 - a^2) = b^2(a^2 + c^2 - b^2)$ factors as $(a-b)(a+b)(a^2 + b^2 - c^2) = 0$, so $a = b$ or $C = 90°$ (never both, since $a + b > 0$). $a = b$ gives area $10\sqrt{6}$; $C = 90°$ with $a + b = 14$ gives $ab = 48$, area 24: the larger is $10\sqrt{6}$
18	D	mean-shift Diophantine search	shift to the mean $m = S/3$: with $x = 3a - S$ etc., $x + y + z = 0$ and $x^2 + y^2 + z^2 = 9Q - 3S^2 = 2616$, i.e. $x^2 + xy + y^2 = 1308$ after eliminating $z = -(x+y)$; a short integer search on this ellipse gives a single triple up to order: $(a, b, c) = (-12, -2, 12)$
19	E	iterate and substitute	for $\lvert x\rvert > 2$, $f(x) - x = (x-2)(x+1) > 0$ when $x > 2$ (and symmetrically for $x < -2$), so every solution lies in $[-2, 2]$; write $x = 2\cos\theta$, then $f(2\cos\theta) = 2\cos 2\theta$, so the n-fold iterate is $2\cos(2^3\theta)$: solving $\cos(2^3\theta) = \cos\theta$ on $[0, \pi]$ gives $\theta = \frac{2k\pi}{2^3-1}$ (4 values) or $\theta = \frac{2k\pi}{2^3+1}$ (5 values), overlapping only at $\theta = 0$: $8 = 2^3$ solutions
20	C	divisor-chain recurrence	$f(n) = \sum_{d \mid n, d<n} f(d)$ depends only on the multiset of exponents in n's prime factorisation; a direct count over $n = 1, \ldots, 100$ gives 4 values with $f(n) = 2$

Set 1, Paper 2

Q	Ans	Circuit	Reflex
1	D	range symmetry	substitute $x = 5$: $f(5) + f(-7) = -8$, so $f(-7) = -8 - (-3) = -5$
2	A	counterexample	x^4 has a minimum at 0, where $f'' = 0$ but the concavity does not change
3	A	statement testing	test each statement on its own; a single counterexample kills a 'for all'
4	D	end behaviour	end behaviour: even degree has equal signs at $\pm\infty$, odd degree opposite; then compare with the sign of $p(0)$
5	E	inscribed angle	central angle is double; angles in the same segment are equal; opposite arc gives the supplement

Q	Ans	Circuit	Reflex
6	F	AM-GM	AM-GM: $\sqrt{xy} \le \frac{x+y}{2} = 3$; $x^2 + y^2 \ge \frac{(x+y)^2}{2}$
7	G	square the middle	square the middle; blocks of a GP are earlier blocks times r^k
8	A	larger set	$x^2 > 4$ is $x > 2$ or $x < -2$: the larger set
9	D	compartmentalise and negate	compartmentalise: flip every quantifier, then flip the final claim
10	C	$\bigstar + \triangle = 2a$	add the two arguments: they must sum to 6 with equal values
11	D	touch or cross	odd multiplicity crosses, even multiplicity touches
12	B	statement testing	test each statement on its own; a single counterexample kills a 'for all'
13	B	$\bigstar + \triangle = 2a$	point symmetry about $(3, 0)$, the midpoint of the interval, kills the integral
14	A	proof audit	the argument only covers $x \ge 1$; $x = \frac{1}{2}$ is a counterexample
15	H	shared axis, given root	$f(6) = f(4)$ forces the axis of symmetry to $x = \frac{6+4}{2} = 5$ regardless of a and c; reflecting the given root 4 in that axis gives the other root 6 (equal to it only if $r = 5$, which is excluded here)
16	B	compose two reflections	point symmetry about $(-3, -4)$ plus line symmetry about $x = 0$ compose: shifting by $6 = 2(0 - -3)$ sends $f(x) \mapsto 2b - f(x)$, and shifting by 12 returns $f(x)$ unchanged. From 0 to -30 is -5 such half-shifts, an odd count, so $f(-30) = 2b - f(0) = -9$
17	F	Vieta on f'	sketch the sign of f' across each root: it changes sign at a simple root and holds its sign through a repeated one
18	C	range symmetry	$f(x) + f(2a - x) = 2b$ is point symmetry about $(0, 1)$, so the range is symmetric about 1: minimum $= 2b - M = -4$; the symmetry sends the maximiser $x = 3$ to $x = 2a - c = -3$: $-4 + -3 = -7$
19	C	range symmetry	$f(x) + f(2a - x) = 2b$ is point symmetry about $(0, -2)$, so the range is symmetric about -2: minimum $= 2b - M = -9$; the symmetry sends the maximiser $x = -3$ to $x = 2a - c = 3$: $-9 + 3 = -6$
20	G	shared axis, given root	$f(-1) = f(-2)$ forces the axis of symmetry to $x = \frac{-1+-2}{2} = -\frac{3}{2}$ regardless of a and c; reflecting the given root -4 in that axis gives the other root 1 (equal to it only if $r = -\frac{3}{2}$, which is excluded here)