TMUA9.
MOCK ZETA.

One set, Paper 1 and Paper 2, TMUA format.
Free. No NOIR PREMIERE required.
Official time 75 minutes. NOIR target: 15.

TMUA9.

How to use this mock

One set: a Paper 1 (Applications of Mathematical Knowledge) and a Paper 2 (Mathematical Reasoning), twenty multiple-choice questions each, in the TMUA format: pick one letter, no working marked, no calculator.

The point. The official paper gives you 75 minutes. A NOIR-trained student should be done in 15. Every question is one reaction circuit with nothing else in the way, named in cyan at the end of the question, except the last few in each paper, which chain two or three ideas together and are left unnamed on purpose: that is the part of the paper built to feel like a real Q15-20.

The rule. Set the timer for 15 minutes per paper. Stop when it rings, whatever is left is a miss. Mark from the key at the back: every answer comes with the circuit and the one-line reflex.

Verification. Every numeric answer was recomputed from the underlying integral, root, or coordinate geometry by a computer algebra system, independently of the shortcut; every logic answer was checked by brute force where a predicate could be evaluated.

Want more of this. This is one of eight free sets in the TMUA9 Question Bank. The full ten-set TMUA9. Papers, the DELUXE rehearsal, and NOIR PREMIERE go further: `https://tmua9.com/noir-premiere`.

Set 1 Paper 1 Applications of Mathematical Knowledge

TMUA9. PAPERS

20 questions. Official time 75 minutes. NOIR target: 15 minutes. Every question is one reaction circuit, named in the margin. Answer on the grid at the end of the set.

1 A sequence $a_1, a_2, a_3, \ldots$ satisfies $a_1 = 3$ and $a_n + a_{n+1} = 4n + 3$ for every positive integer n. What is a_7? [pair the recurrence]

A -15 **B** 5 **C** 12 **D** 14 **E** 15 **F** 19

2 The curve $y = -9x^3 + bx^2 + cx + d$ touches the line $y = k$ at $x = -1961$ and crosses it at $x = -1958$. What is the area of the region enclosed between the curve and the line? [★^{4}/12]

A $\frac{243}{8}$ **B** $\frac{729}{16}$ **C** $\frac{243}{4}$
D $\frac{247}{4}$ **E** $\frac{251}{4}$ **F** 243

3 A sequence $a_1, a_2, a_3, \ldots$ satisfies $a_1 = 3$ and $a_n + a_{n+1} = -2n + 1$ for every positive integer n. What is a_7? [pair the recurrence]

A -8 **B** -5 **C** -4 **D** -3 **E** -2 **F** -1

4 Tangents are drawn to $y = 3x^2 + bx + c$ at the points where $x = -1961$ and $x = -1951$. The region enclosed by the two tangents and the curve has area 250. What is the area between the curve and the chord joining the points of tangency? [$S{:}T$]

A 125 **B** 250 **C** 499 **D** 500 **E** 501 **F** 3000

5 What is the term independent of x in the expansion of $\left(x^1 - \frac{1}{x^3}\right)^{12}$? [multinomial]

A -264 **B** -220 **C** 0 **D** 66 **E** 220 **F** 495

6 The curve $y = -x^2 + bx + c$ has axis of symmetry $x = 1731$ and meets the line $y = k$ at $x = 1729$ and $x = 1733$. The region enclosed by the curve, the line $y = k$ and the vertical line $x = m$ ($m > 1733$) has half the area of the region enclosed by the curve and $y = k$. What is m? [$\sqrt{3}$]

A $\frac{1731+2\sqrt{3}}{3}$ **B** $1731 + 2\sqrt{2}$ **C** $1731 + 2\sqrt{3}$
D 1735 **E** $1733 + 2\sqrt{3}$ **F** 1737

7 The equation $x^3 - 5x^2 - 2x + 24 = 0$ has three real roots. What is the product of the roots? [Vieta]

A -48 **B** -25 **C** -24 **D** -23 **E** 24 **F** 576

8 A cubic curve $y = 2x^3 + ax^2 + bx$ (where a and b are constants) has a local maximum at $x = -4$ and a local minimum at $x = 3$. What is b? [Vieta on f']

A -72 **B** -70 **C** -48 **D** -36 **E** -24 **F** -18

9 The graph of $y = g(x)$ is obtained from $y = f(x)$ by a reflection in the y-axis followed by a translation of 4 units in the negative x-direction. The graph of $y = h(x)$ is obtained by the same two transformations in the opposite order. Which condition is necessary and sufficient for $g(x) = h(x)$ for all x? [period]

A $f(x) = f(x+4)$ **B** $f(x) = f(x+8)$ **C** $f(x) = f(x+16)$
D $f(x) = f(-x)$ **E** $f(x) = f(8-x)$ **F** $f(x) = f(4-x)$

10 The tangent to the curve $y = x^3 + bx$ (where b is a constant) at the point where $x = -2$ passes through the point $(-6, -62)$, which does not lie on the curve. What is the x-intercept of this tangent? [ext. tangent]

A $-\frac{96}{13}$ **B** -6 **C** $-\frac{64}{13}$
D -2 **E** $-\frac{16}{13}$ **F** $\frac{4\sqrt{13}}{13}$

11 The curve $y = -x^3 + bx^2 + cx + d$ meets the line $y = k$ at $x = -2048$, $x = -2047$ and $x = -2045$. What is the area of the region enclosed between the curve and the line for $-2047 \le x \le -2045$? [lobes]

A $\frac{5}{12}$ **B** $\frac{8}{9}$ **C** $\frac{8}{3}$
D $\frac{37}{12}$ **E** $\frac{11}{3}$ **F** 16

12 A cubic curve $y = x^3 + ax^2 + bx$ (where a and b are constants) has a local maximum at $x = -3$ and a local minimum at $x = 3$. What is a? [Vieta on f']

A -2 **B** -1 **C** 0 **D** 1 **E** 2

13 The curve $y = \frac{1}{2}x^4 + 1$ and the lines $x = 2$ and $y = 1$ enclose a region. The rectangle with vertices $(0,1)$, $(2,1)$, $(2,9)$ and $(0,9)$ is divided by the curve into two parts. What is the ratio of the larger part to the smaller? [n:1]

A 2 : 1 **B** 3 : 1 **C** 4 : 1 **D** 5 : 1 **E** 6 : 1

14 What is the sum of all solutions of the equation $\cos 2x = -\frac{3}{2}\cos x - \frac{1}{2}$ in the interval $0 \le x \le 2\pi$? [root-pair sum]

A π **B** 2π **C** 3π **D** 4π **E** 6π **F** 36π

15 Functions f and g are defined for all real x by $f(x) = 1x|x|$ and

$$g(x) = \begin{cases} a(x+3) & x < -3 \\ 0 & -3 \le x \le 3 \\ a(x-3) & x > 3 \end{cases}$$

where $a > 0$ is a constant. Let $h(x) = \int_0^x (g(u) - f(u))\,du$. The largest value of a for which h attains a local maximum or a local minimum value at exactly one value of x is k. When $a = k$, what is $k + h(4)$? [piecewise extremum]

A $-\frac{46}{3}$ **B** $-\frac{10}{3}$ **C** $-\frac{5}{2}$
D 12 **E** 15 **F** $\frac{82}{3}$

16 Two circles, of radii 25 and 4, touch one another and both touch a common straight line ℓ, on the same side of ℓ. Circles $C_1, C_2, C_3, \ldots$ are then inscribed in the successive gaps: each C_n touches ℓ, touches the circle of radius 25, and touches C_{n-1} (with C_0 the circle of radius 4). For how many values of n does C_n have radius greater than $\frac{1}{25}$? [circle chain]

A 22 **B** 23 **C** 33 **D** 44 **E** 88 **F** 132

17 Triangle ABC has $BC = a$, $CA = b$, $AB = c = 6$, and $a\cos A = b\cos B$. Given also that $a + b = 7$, what is the largest possible area of the triangle? [two-branch cosine rule]

A $\frac{13}{4}$ **B** $\frac{3\sqrt{13}}{2}$
C $\frac{4+3\sqrt{13}}{2}$ **D** $\frac{13+6\sqrt{13}}{4}$
E 21 **F** $9\sqrt{13}$

18 Three distinct integers $a < b < c$ satisfy

$$a + b + c = -7, \qquad a^2 + b^2 + c^2 = 219.$$

What is the middle of the three? [mean-shift Diophantine]

A -13 **B** -7 **C** -3 **D** -2 **E** -1 **F** 7

19 $f(x) = x^2 - 2$. How many real solutions does $\underbrace{f(f(\cdots f}_{3}(x)\cdots)) = x$ have? [iterate + substitution]

A 4 **B** 6 **C** 7 **D** 8 **E** 9 **F** 32

20 For each integer $n \geq 1$ let $f(n)$ be the number of lists of *distinct* positive integers which begin with 1, end with n, and in which every term except the last divides the term after it. (For example $f(1) = 1$, $f(2) = 1$ and $f(6) = 3$, the three lists being $1, 6$ and $1, 2, 6$ and $1, 3, 6$.) How many integers n with $1 < n \leq 100$ satisfy $f(n) = 44$? [divisor chain]

A 1 **B** 2 **C** 3 **D** 4 **E** 6 **F** 36

Set 1 Paper 2 Mathematical Reasoning

TMUA9. PAPERS

20 questions. Official time 75 minutes. NOIR target: 15 minutes. Every question is one reaction circuit, named in the margin. Answer on the grid at the end of the set.

1 Which of the following is the negation of the statement "For all real x, if $x > 4$ then $x^2 > 16$."? [negate]

A For all real x, if $x > 4$ then $x^2 \leq 16$.
B For all real x, if $x \leq 4$ then $x^2 \leq 16$.
C There is a real x with $x > 4$ and $x^2 \leq 16$.
D There is a real x with $x \leq 4$ and $x^2 > 16$.
E There is a real x with $x > 4$ or $x^2 \leq 16$.
F For all real x, if $x^2 \leq 16$ then $x \leq 4$.

2 A function f satisfies $f(x) + f(-6-x) = -8$ for every real x. Given that $f(1) = -3$, what is $f(-7)$? [range symmetry]

A -60 **B** -15 **C** -11 **D** -5 **E** -3 **F** 3

3 Which of the following is a sufficient condition for $\int_1^5 f(x)\,dx = 0$? [★ + △]

A $f(-x) = -f(x)$ for all x
B $f(1) = f(5) = 0$
C $f(1) = -f(5)$
D $f(3+x) = f(3-x)$ for all x
E $f(3+x) = -f(3-x)$ for all x
F $f(3) = 0$

4 x and y are positive real numbers with $x + y = 4$. Which of the following must be true?

I $\frac{1}{x} + \frac{1}{y} \geq 1$
II $xy \geq 4$
III $x^2 + y^2 \geq 8$ [AM-GM]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

5 Which of the following statements is/are true?

I A triangle with sides 5, 8 and 12 exists
II $x^2 + 2x + 1 > 0$ for all real x
III Every quartic polynomial has at least one real root [statements]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only

G II and III only
H I, II and III

6 How many solutions does the equation $\sin(2x) = -\frac{1}{2}$ have in the interval $0 \leq x \leq 2\pi$? [seasons]

A 2 **B** 3 **C** 4 **D** 5 **E** 8

7 Which of the following statements is/are true?
I The graph of $y = (x-1)^2(x+2)$ touches the x-axis at $x = 1$
II $x^2 > 3$ is a sufficient condition for $x > \sqrt{3}$
III If $f'(c) = 0$ then f has a turning point at $x = c$ [statements]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

8 Let $f(x) = ax^2 + bx + c$ with $a > 0$ and $f(-1) < 0$. Which of the following must be true?
I the roots of $f(x) = 0$ lie on either side of $x = -1$
II f has a minimum value less than 0
III $f(0) > 0$ [statements]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

9 A, B and P lie on a circle with centre O, and $\angle APB = 120°$. Which of the following must be true?
I the reflex angle AOB is $240°$
II $\angle AQB = 120°$ for every point Q on the circle other than A and B
III $\angle OAB = 30°$ [inscribed]

A none of them
B I only
C II only
D III only
E I and II only
F I and III only
G II and III only
H I, II and III

10 A function f satisfies $f(x) + f(-4-x) = 4$ for every real x. Given that $f(3) = 2$, what is $f(-7)$? [range symmetry]

A -2 **B** 1 **C** 2 **D** 3 **E** 6 **F** 12

11 The equation $4x^3 + 30x^2 + 72x = c$ has three distinct real solutions. Which of the following is the complete set of possible values of c? [root band]

A $c < -56$ or $c > -54$ **B** $54 < c < 56$ **C** $c < -56$
D $-56 \leq c \leq -54$ **E** $c > -54$ **F** $-56 < c < -54$

12 For real numbers x, which of the following describes the condition $x(x-2) > 0$ in relation to the condition $x > 2$? [larger set]

A necessary but not sufficient

B sufficient but not necessary

C necessary and sufficient

D neither necessary nor sufficient

13 Consider the statements P: "$x^2 = 1$" and Q: "$x = 1$". Which of the following is true? [larger set]

A P is necessary but not sufficient for Q

B P is sufficient but not necessary for Q

C P is necessary and sufficient for Q

D P is neither necessary nor sufficient for Q

14 $b_1, b_2, b_3, \ldots$ is a geometric progression with common ratio r ($r \neq 0, r \neq \pm 1$) and S_n is the sum of its first n terms. Which of the following must be true?

I $b_4 + b_6 = 2b_5$

II $\dfrac{S_2}{S_1} = r^1$

III $b_4 b_7 = b_5 b_6$ [middle^2]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

15 $f(x) = ax^2 + bx + c$ where $a \neq 0$, and $f(-1) = f(-6)$. Given also that $f(5) = 0$, which of the following must be true?

I $f(0) = 0$

II $f(-12) = 0$

III the sum of the roots of $f(x) = 0$ is -7 [axis + root]

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

16 A function f is defined for all real x and satisfies $f(x) + f(-4-x) = 2$ and $f(1+x) = f(1-x)$ for every real x. Given that $f(3) = 0$, what is $f(21)$? [reflect twice]

A 0 **B** 1 **C** 2 **D** 4 **E** 6 **F** 12

17 A function f satisfies $f'(x) = 3x^3 + 9x^2 - 3x - 9$ for all real x. Which of the following must be true?

I f has a local maximum at $x = 1$
II f has a local minimum at $x = 1$
III the equation $f'(x) = 0$ has a root strictly between -1 and 1 [Vieta on f']

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

18 A function f, continuous on the whole real line, satisfies $f(x) + f(-2 - x) = -8$ for every real x. The maximum value of f is 8, attained only at $x = -2$. What is (the minimum value of f) + (the value of x at which that minimum occurs)? [range symmetry]

A -48 **B** -32 **C** -16 **D** 0 **E** 4 **F** 6

19 A function f satisfies $f'(x) = \frac{x^3}{2} - x^2 - \frac{x}{2} + 1$ for all real x. Which of the following must be true?

I f has a local minimum at $x = 1$
II f has a local maximum at $x = -1$
III the equation $f'(x) = 0$ has a root strictly between -1 and 1 [Vieta on f']

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

20 A function f is defined for all real x and satisfies $f(x) + f(-2 - x) = 6$ and $f(-3 + x) = f(-3 - x)$ for every real x. Given that $f(2) = 1$, what is $f(14)$? [reflect twice]

A -3 **B** 1 **C** 5 **D** 6 **E** 20 **F** 30

Answer key

Each answer with the circuit that fires and the one-line reflex. Mark, then for every miss write the circuit name next to the question and redo it the NOIR way.

Set 1, Paper 1

Q	Ans	Circuit	Reflex
1	E	paired recurrence	unwind $a_{n+1} = (4n+3) - a_n$ from $a_1 = 3$ down to $a_7 = 15$
2	C	$\bigstar^4/12$	tangent to a cubic: $S = \frac{\lvert a\rvert}{12}\bigstar^4 = \frac{243}{4}$
3	D	paired recurrence	unwind $a_{n+1} = (-2n+1) - a_n$ from $a_1 = 3$ down to $a_7 = -3$
4	D	$S : T = 2 : 1$	$S : T = 2 : 1$, so $S = 500$
5	B	multinomial	$p(n-k) = qk$ gives $k = 3$: $\binom{12}{3}(-1)^3 = -220$
6	C	$\sqrt{3}$ law	$\sqrt{3}$ law: $m = \text{axis} + \sqrt{3}\,t = 1731 + 2\sqrt{3}$
7	C	Vieta	Vieta on the cubic, roots never needed: -24
8	A	Vieta on f'	$f'(x) = 6x^2 + 6x - 72$ must have roots $-4, 3$: Vieta on the derivative gives $a = 3, b = -72$
9	B	reflect-translate period	$g(x) = f(-x-4), h(x) = f(-x+4)$: equal iff f has period 8
10	E	external tangent	the tangent at $x = -2$ has gradient $3t^2 + b$ and y-intercept $-2t^3 = 16$; passing through $(-6, -62)$ forces gradient 13, so $b = 1$ and the x-intercept is $\dfrac{2t^3}{3t^2+b} = -\frac{16}{13}$
11	C	unequal lobes	$S_\bigstar = \frac{\lvert a\rvert}{6}\bigstar^3(\frac{\bigstar}{2} + \triangle)$ with $\bigstar = 2, \triangle = 1$: $\frac{8}{3}$
12	C	Vieta on f'	$f'(x) = 3x^2 - 27$ must have roots $-3, 3$: Vieta on the derivative gives $a = 0$, $b = -27$
13	C	$n : 1$ box	corner at the stationary or inflection point: $n : 1$ with $n = 4$
14	C	root-pair sum	with $u = \cos x$: $2u^2 - (-\frac{3}{2})u - (\frac{1}{2}) = 0$ has roots $u = -1, \frac{1}{4}$; a root with $-1 < u \le 1$ contributes a pair of x-solutions summing to 2π (for $u = 1$ that pair is $x = 0$ and $x = 2\pi$, the trap most people miss), but $u = -1$ contributes only the single solution $x = \pi$: total 3π
15	B	piecewise extremum (parameter search)	$h' = g - f$ is odd and always flips sign at $x = 0$ (that extremum never depends on a). On $x \ge 3$, $h' = a(x-3) - 1x^2$ is a downward parabola peaking at $\frac{a^2}{4c} - aw$: the peak first touches 0 when $a = 4cw = k = 12$, and past that two more sign changes appear on each side by symmetry. $h(4) = -\frac{46}{3}$, so $k + h(4) = -\frac{10}{3}$
16	A	circle-chain counting	if circles of radii p, q touch a line on the same side and touch each other, $\frac{1}{\sqrt{w}} = \frac{1}{\sqrt{p}} + \frac{1}{\sqrt{q}}$ for a circle w wedged between them and the line; here $\frac{1}{\sqrt{r_n}} = \frac{1}{\sqrt{4}} + \frac{n}{\sqrt{25}}$, an arithmetic sequence, so $r_n > \frac{1}{25}$ for $n < \frac{45}{2} \approx 22.50$: 22 values
17	B	two-branch cosine rule	clear denominators in $a\cos A = b\cos B$: $a^2(b^2+c^2-a^2) = b^2(a^2+c^2-b^2)$ factors as $(a-b)(a+b)(a^2+b^2-c^2) = 0$, so $a = b$ or $C = 90°$ (never both, since $a + b > 0$). $a = b$ gives area $\frac{3\sqrt{13}}{2}$; $C = 90°$ with $a + b = 7$ gives $ab = \frac{13}{2}$, area $\frac{13}{4}$: the larger is $\frac{3\sqrt{13}}{2}$
18	E	mean-shift Diophantine search	shift to the mean $m = S/3$: with $x = 3a - S$ etc., $x + y + z = 0$ and $x^2 + y^2 + z^2 = 9Q - 3S^2 = 1824$, i.e. $x^2 + xy + y^2 = 912$ after eliminating $z = -(x+y)$; a short integer search on this ellipse gives a single triple up to order: $(a, b, c) = (-13, -1, 7)$
19	D	iterate and substitute	for $\lvert x\rvert > 2$, $f(x) - x = (x-2)(x+1) > 0$ when $x > 2$ (and symmetrically for $x < -2$), so every solution lies in $[-2, 2]$; write $x = 2\cos\theta$, then $f(2\cos\theta) = 2\cos 2\theta$, so the n-fold iterate is $2\cos(2^3\theta)$: solving $\cos(2^3\theta) = \cos\theta$ on $[0, \pi]$ gives $\theta = \frac{2k\pi}{2^3-1}$ (4 values) or $\theta = \frac{2k\pi}{2^3+1}$ (5 values), overlapping only at $\theta = 0$: $8 = 2^3$ solutions
20	C	divisor-chain recurrence	$f(n) = \sum_{d\mid n, d<n} f(d)$ depends only on the multiset of exponents in n's prime factorisation; a direct count over $n = 1, \ldots, 100$ gives 3 values with $f(n) = 44$

Set 1, Paper 2

Q	Ans	Circuit	Reflex
1	C	compartmentalise and negate	compartmentalise: flip every quantifier, then flip the final claim
2	D	range symmetry	substitute $x = 1$: $f(1) + f(-7) = -8$, so $f(-7) = -8 - (-3) = -5$
3	E	$\bigstar + \triangle = 2a$	point symmetry about $(3, 0)$, the midpoint of the interval, kills the integral

Q	Ans	Circuit	Reflex
4	F	AM-GM	AM-GM: $\sqrt{xy} \le \frac{x+y}{2} = 2$; $x^2 + y^2 \ge \frac{(x+y)^2}{2}$
5	B	statement testing	test each statement on its own; a single counterexample kills a 'for all'
6	C	seasons	$\sin(2x)$ completes 2 full oscillations on $[0, 2\pi]$, two solutions each
7	B	statement testing	test each statement on its own; a single counterexample kills a 'for all'
8	E	statement testing	sketch the parabola from the given facts; look for a counterexample before believing a 'must'
9	F	inscribed angle	central angle is double; angles in the same segment are equal; opposite arc gives the supplement
10	C	range symmetry	substitute $x = 3$: $f(3) + f(-7) = 4$, so $f(-7) = 4 - (2) = 2$
11	F	root band	stationary values -54 and -56; the horizontal line $y = c$ must lie strictly between them
12	A	larger set	$x(x-2) > 0$ means $x < 0$ or $x > 2$: the larger set
13	A	larger set	draw the two sets; the sufficient condition is the smaller set
14	D	square the middle	square the middle; blocks of a GP are earlier blocks times r^k
15	G	shared axis, given root	$f(-1) = f(-6)$ forces the axis of symmetry to $x = \frac{-1+-6}{2} = -\frac{7}{2}$ regardless of a and c; reflecting the given root 5 in that axis gives the other root -12 (equal to it only if $r = -\frac{7}{2}$, which is excluded here)
16	C	compose two reflections	point symmetry about $(-2, 1)$ plus line symmetry about $x = 1$ compose: shifting by $6 = 2(1 - -2)$ sends $f(x) \mapsto 2b - f(x)$, and shifting by 12 returns $f(x)$ unchanged. From 3 to 21 is 3 such half-shifts, an odd count, so $f(21) = 2b - f(3) = 2$
17	C	Vieta on f'	sketch the sign of f' across each root: it changes sign at a simple root and holds its sign through a repeated one
18	C	range symmetry	$f(x) + f(2a - x) = 2b$ is point symmetry about $(-1, -4)$, so the range is symmetric about -4: minimum $= 2b - M = -16$; the symmetry sends the maximiser $x = -2$ to $x = 2a - c = 0$: $-16 + 0 = -16$
19	A	Vieta on f'	sketch the sign of f' across each root: it changes sign at a simple root and holds its sign through a repeated one
20	C	compose two reflections	point symmetry about $(-1, 3)$ plus line symmetry about $x = -3$ compose: shifting by $-4 = 2(-3 - -1)$ sends $f(x) \mapsto 2b - f(x)$, and shifting by -8 returns $f(x)$ unchanged. From 2 to 14 is -3 such half-shifts, an odd count, so $f(14) = 2b - f(2) = 5$