TMUA9. ANALYSIS.

Cambridge Assessment Admissions Testing

TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION

PAPER 1

SPECIMEN Time: 75 minutes

Additional Materials: Answer sheet

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one.
You also require a soft pencil and an eraser.

This paper is the first of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only points for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

Any rough work should be done on this question paper. No extra paper is allowed.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

Calculators must **NOT** be used. There is no formulae booklet for this test.

Please wait to be told you may begin before turning this page

This question paper consists of 12 printed pages and 4 blank pages

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* I subbed $y = *$ into this implicit expression even at the cost of multiplying both sides because even then it's simpler relative to subbing in $x = 3y - 1$.

9.

1. The sum of the two values of x that satisfy the simultaneous equations $x - 3y + 1 = 0$ and $3x^2 - 7xy = 5$ is

Don't make the stupendous excuse that it's the same stuff, because it's not. These small nudges to reduce at least one line of computation, ALL compound.

A -8.5

B -7.5

C -1.5

D 3.5

E 4.5

F 5

$3y = x + 1$

$9x^2 - 7x(3y) = 15$ → Write into explicit form before substitution but have LHS be sth substitution required.

$9x^2 - 7x(x+1) = 15$

9.

$2x^2 - 7x - 15 = 0.$

$-1.5 \quad 5$

$(2x+3)(x-5) = 0.$

$2 \quad +3$

$1 \quad -5$

* $\sin^2\theta$ & $\cos\theta$ mixed ⇒ Concentrate into only one, either sine/cosine.

9.

2. The number of solutions in the interval $0 \le \theta \le 4\pi$ of the equation $\sin^2\theta + 3\cos\theta = 3$ is

* Caveat: If you're going to substitute $\cos\theta$ as c, then define domain. Manipulating as variable, you can lose sight that it's in $[-1,1]$.

$-1 \le c \le 1$

A 0

B 1

C 2

D 3

E 4

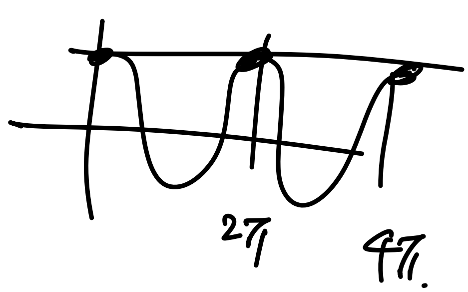
F 5

G 6

$1 - c^2 + 3c = 3.$

$c^2 - 3c + 2 = 0.$

$c = 1.$



9 THE DEBUT MUST BE ULTIMATE.

9. when writing first expression, of where you can write all into one equation/function, you should.

9.

3. The perpendicular bisector of the line segment joining the points $(2, -6)$ and $(5, 4)$ cuts the x-axis at the point with x-coordinate

☆ Much better if you could write into one single expression at first.

$$y = -\frac{2-5}{-6-4}(x-3.5)+(-1)$$

A $\frac{1}{20}$

B $\frac{1}{6}$

C $\frac{1}{3}$

D $\frac{19}{5}$

E $\frac{41}{6}$

$$0 = -\frac{3}{10}(x-3.5)-1$$

$$\frac{3}{10}(x-3.5) = -1$$

$$x-3.5 = -\frac{10}{3}$$

$$x = \frac{7}{2}-\frac{10}{3} = \left(3+\frac{1}{2}\right)-\left(3+\frac{1}{3}\right)$$

What I did & generalised form:

let $P(x_p, y_p)$, $Q(x_q, y_q)$ & l is perpendicular bisector.

Gradient of PQ: $\frac{y_q-y_p}{x_q-x_p}$.

$l \times PQ = -1$ (Gradient product) $\rightarrow$ l gradient $= \frac{-1}{PQ} = -\frac{x_p-x_q}{y_p-y_q}$, l passes midpoint of PQ: $M\left(\frac{x_p+x_q}{2}, \frac{y_p+y_q}{2}\right)$

9. $l: -\frac{x_p-x_q}{y_p-y_q}\left(x-\frac{x_p+x_q}{2}\right)+\frac{y_p+y_q}{2}$

9.

4. The complete set of values of x for which $(x^2-1)(x-2) > 0$ is

A $x < -1,\ 1 < x < 2$

B $x < -1,\ x > 2$

C $-1 < x < 2$

D $x < 1,\ x > 2$

E $-1 < x < 1,\ x > 2$

* you should be able to sketch $(x^2-1)(x-2)$ at this point.

① your immediate response to (Cubic) ≥ 0.

① inequality inclusive of

9. Inequality adds dimensionality to an equation.

→ When seeing equation, ① equate it & solve

②

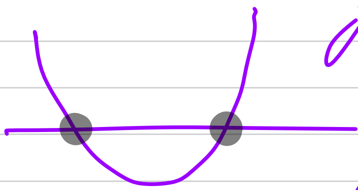
9. Inequality adds dimensionality to equalities.

Let's look more into detail to what this even means.

→ "$x^2-1=0$"

this is just an equation (Algebraic Perspective)

In a Graph's perspective, it is two functions: $\begin{cases} y=x^2-1 \\ y=0 \end{cases}$



$y=x^2-1$ is a parabola $\in$ Curved line $\in$ line = 1 Dimension.

$y=0$ is the x-axis $\in$ straight line $\in$ line = 1 Dimension.

As equating 1 Dimension = 1 Dimension, our interest are the points (0 Dimension) of interest.

What if we change (=) into an inequality?

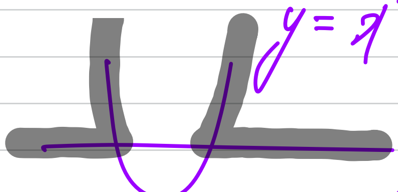
$x^2-1 \geq 0$

let $y=f_1(x) : f_1(x)=x^2-1$

$y=f_2(x) : f_2(x)=0$

cf) Don't think of it as some magical axis. It's just one amongst the infinite lines. Be free from shackles of the x,y axis...😎

+1 Dimension



But here, our interest is the relative position of lines. (1 Dimension)

The basic premise is still to lean towards graphs !?

9.

This question was completely stupid to do graphs though.

5. Given that $y = -\log_{10}(1-x)$ for $x < 1$, find x in terms of y.

$-y = \log_{10}(1-x)$

$10^{-y} = 1-x$

cf) □ try to articulate this from scratch.

A $x = -\frac{1}{\log_{10}(1-y)}$

B $x = 1 + \log_{10} y$

C $x = 1 - \log_{10} y$

D $x = 1 - 10^{-y}$

$x = 1 - 10^{-y}$.

E $x = 10^{-y} - 1$

F $x = 10^{1-y}$

* the premise is... think of graphs always. But here the intuition to discern when to sketch graphs & when not to. NOT BEING ABLE TO SKETCH A GRAPH MUST NOT BE AN EXCUSE.

$f(x) = x^3 + 4cx^2 + x(c+1)^2 - 6.$

* factor theorem

9.

6. It is given that $x + 2$ is a factor of $x^3 + 4cx^2 + x(c+1)^2 - 6$.

The sum of the possible values of c is

$f(-2) = -8 + 16c - 2(c+1)^2 - 6.$

A -10

B -6

C 0

$= 16c - 14 - 2c^2 - 4c - 2$

D 6

E 10

$= -2c^2 + 12c - 16.$

Remainder theorem

$f(x) = (x-a)Q(x) + R, \; R \in \mathbb{R}$

$f(a) = (a-a)Q(a) + R.$

$= -2(c^2 - 6c + 8)$

$f(a) = R.$

□ IS this statement true? : factor theorem is sufficient but not necessary for remainder theorem?

* factor theorem: if $f(a) = 0$, then $(x-a)$ is a factor of $f(x)$.

Small Logic Play. (#6)

* let P, Q be truth sets.

P : Remainder Theorem

Q : Factor Theorem

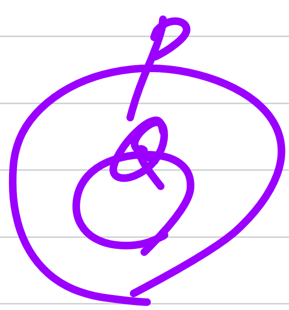
Galleria Logic's truth set & proposition

→ Then propositions are

p : If you divide polynomial $f(x)$ by linear term $(x-c)$, then the remainder is equal to $f(c)$ (= evaluation of $f(x)$ at $x=c$)

q : $(x-a)$ is a factor of polynomial $f(x)$ if and only if (=abv. as iff) $f(a)=0$.

$$\left.\begin{array}{l} q \to p \\ p \not\to q \end{array}\right\}$$



So P is the larger set.

☆ When selecting balls type question, time matters! (Is it all taken out at once, or sequently?)

9.

7. A bag contains n red balls, n yellow balls, and n blue balls.

One ball is selected at random and not replaced.

A second ball is then selected at random and not replaced.

Each ball is equally likely to be chosen.

The probability that the two balls are **not** the same colour is

Complementary set

Very common approach.

e.g.) $\boxed{} \rightarrow$ 5 same oranges. & 2 same apples. (So total 7 people) box $A_1, A_2, \cdots A_7$. and one fruit goes to one box.

Q1) Total arranges of oranges & apples.

Q2) Cases where the apples don't sit together.

↗ "Galleria: Logic."

A $\dfrac{n-1}{3n-1}$

B $\dfrac{2n-2}{3n-1}$

C $\dfrac{2n}{3n-1}$

D $\dfrac{(n-1)^3}{27(3n-1)^3}$

E $\dfrac{3(n-1)}{3n-1}$

F $\dfrac{n^3}{27(3n-1)^3}$

$$1-\left\{\frac{n}{3n}\times\frac{n-1}{3n-1}\times \underbrace{3}_{\text{3 colours}}\right\}$$

$$= 1-\frac{n-1}{3n-1}$$

$$= \frac{3n-1-n+1}{3n-1}$$

9. * Check whether or not, order is taken into account.

* Consider solving via complementary sets (= reverse engineering)

↓ literally 50/50 with forward engineering.

0. 7. "Galleria: Statistics/Probability":

Why? TMUA9. frame) There are 7 chairs that are different, and I'm choosing 2 chairs for same apples. ⊕ the rest 5 seats are just filled up by oranges.

Q1) Total Cases: $\underline{{}_7C_2 \times {}_5C_5} = \frac{7!}{2!5!} \times \frac{5!}{0!5!}$

$= \frac{7 \cdot 6}{2 \cdot 1} \times 1 = 21.$

* you must check homogeneity with same group's elements.

(Why?

cf) if Apples/oranges each of them weren't the same, total arrangement would just simply be $7!$. $(=5040)$.

negation!

Q2) Total Cases − (the opposite).

Visual image: U (Universal set = total set) [Adjacent Apples] •

negated (complete opposite) of what we want

What we're looking for → so (Total Arrangement) − (Cases when the apples are adjacent.)

they sit in pairs of $A_{n-1} A_n$. $(n \in \mathbb{N}, n \leq 7)$

$= {}_7C_2 - \underline{6}$

$= 21 - 6 = 15$

$A_1 \ A_2 \ A_3 \ A_4 \ A_5 \ A_6 \ A_7$

★ Same base / Same exponent → team them up!

9

8. Given that $a^x b^{2x} c^{3x} = 2$, where a, b, and c are positive real numbers, then $x =$

A $\log_{10}\left(\frac{2}{a+2b+3c}\right)$

B $\frac{\log_{10} 2}{\log_{10}(a+2b+3c)}$

C $\frac{2}{\log_{10}(a+2b+3c)}$

D $\frac{2}{a+2b+3c}$

E $\log_{10}\left(\frac{2}{ab^2c^3}\right)$

F $\frac{\log_{10} 2}{\log_{10}(ab^2c^3)}$

G $\frac{2}{\log_{10}(ab^2c^3)}$

H $\frac{2}{ab^2c^3}$

$(ab^2c^3)^x = 2.$

$a^x = b$

$\log_a a^x (= x) = \log_a b$

$x = \log_{ab^2c^3} 2$

$\log_a b = \frac{\log_c b}{\log_c a}$

$= \frac{\log 2}{\log_{10} ab^2c^3}$

↗ TMUA9. NOZR PREMZERE.

★ Vieta's formulae

let $ax^2+bx+c = a(x-\alpha)(x-\beta)$

$-a(\alpha+\beta) = \frac{b}{a}$, $a\alpha\beta = c$ → $\begin{cases} \alpha+\beta = -\frac{b}{a} \\ \alpha\beta = \frac{c}{a} \end{cases}$

9

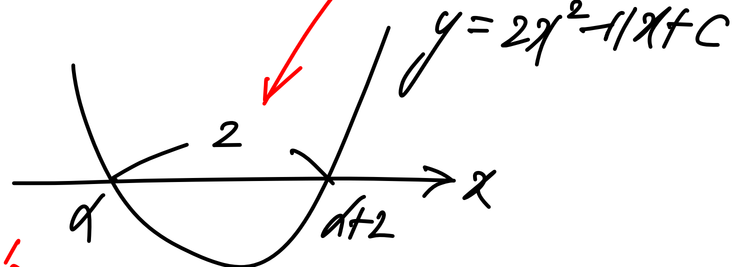
9. The roots of the equation $2x^2 - 11x + c = 0$ differ by 2. The value of c is

A $\frac{105}{8}$

B $\frac{113}{8}$

C $\frac{117}{8}$

D $\frac{119}{8}$



Vieta's formula → $\alpha + (\alpha+2) = \frac{11}{2}$

$2\alpha = \frac{11-4}{2} = \frac{7}{2}.$

$\alpha = \frac{7}{4}$

$\frac{c}{2} = \alpha(\alpha+2) = \frac{7}{4} \cdot \frac{15}{4} = \frac{105}{16}$, $c = \frac{105}{8}$

[Turn over

★ Reflect $y=f(x)$ by $y=k$: Substitute $(2y-k)$ into y.

★ Translate by ★ units in the positive x direction: $(x-k)$ into x.

9.

10. The curve $y = \cos x$ is reflected in the line $y = 1$ and the resulting curve is then translated by $\frac{\pi}{4}$ units in the positive x-direction. The equation of this new curve is

$$y = \cos x \xrightarrow[x:\,x-\frac{\pi}{4}]{y:\,2-y} 2-y = \cos\left(x-\frac{\pi}{4}\right)$$

A $\quad y = 2 + \cos\left(x + \frac{\pi}{4}\right)$

B $\quad y = 2 + \cos\left(x - \frac{\pi}{4}\right)$

C $\quad y = 2 - \cos\left(x + \frac{\pi}{4}\right)$

D $\quad y = 2 - \cos\left(x - \frac{\pi}{4}\right)$

$$y = 2 - \cos\left(x - \frac{\pi}{4}\right).$$

66 Galleria: Function Transformation. 99

★ Very common to omit it.

★ When substituting by introducing new variable, MUST MUST DEFINE NEW DOMAIN.

9.

11. The sum of the roots of the equation $2^{2x} - 8 \times 2^x + 15 = 0$ is

let $2^x = ★.$ $(★>0)$

$★^2 - 8★ + 15 = 0$

$(★-3)(★-5) = 0.$

$★ = 3, 5$

$\therefore 2^x = 3, 5$

$x = \log_2 3, \log_2 5$

$\Sigma x \rightarrow \log_2(3\times5) = \log_2 15 = \dfrac{\log_{10} 15}{\log_{10} 2}$

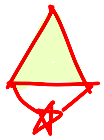
A $\quad 3$

B $\quad 8$

C $\quad 2\log_{10} 2$

D $\quad \log_{10}\left(\frac{15}{4}\right)$

E $\quad \dfrac{\log_{10} 15}{\log_{10} 2}$



· area of equilateral = $\frac{\sqrt{3}}{4}☆^2$.

9.

12. The cross-section of a triangular prism is an equilateral triangle with side $2x$ cm. The length of the prism is d cm.

Again, like #3, Debut must be ultimate.

• write it in one go, not

$T = ☆$

$T = △$

~~step~~ so

$△ = ☆$.

Let the total surface area of the prism be T cm^2. Given that the volume of the prism is T cm^3, which one of the following is an expression for d in terms of x?

$$\text{Area} = \frac{\sqrt{3}}{4}(2x)^2 = \sqrt{3}x^2$$

total surface area   total Volume.

$$T = (2x)d \times 3 + 2\times\sqrt{3}x^2 = \sqrt{3}x^2 \times d$$

$$\times \frac{1}{x}$$

$$6d + 2\sqrt{3}x = \sqrt{3}xd.$$

$$2\sqrt{3}x = (\sqrt{3}x - 6)d$$

A $\quad \dfrac{x}{2x-3}$

B $\quad \dfrac{3x}{3x-2\sqrt{3}}$

C $\quad \dfrac{2x}{x-4\sqrt{3}}$

D $\quad \dfrac{2x}{x-2\sqrt{3}}$

E $\quad \dfrac{2x}{x-\sqrt{3}}$

* this is what I mean when you should recognise cues that the examiner gives you.

$$f(x)=0 \xrightarrow{\text{split into}} g(x)-h(x)=0.$$

I recognised the cue that $x^4-4x^3+4x^2$ as a function that is line symmetric

9.

13. How many real roots does the equation $x^4 - 4x^3 + 4x^2 - 10 = 0$ have?

$f(x) = g(x) - h(x)$, where

$$g(x) = x^4-4x^3+4x^2 = x^2(x^2-4x+4) = x^2(x-2)^2.$$

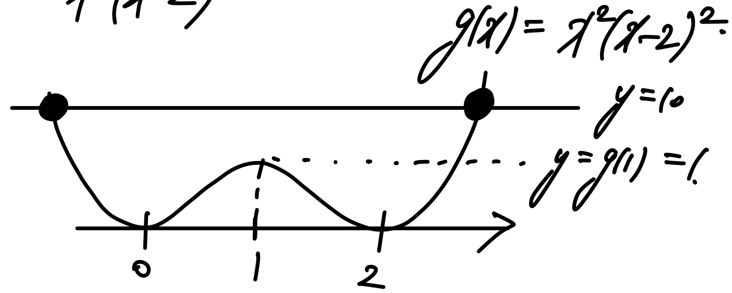
A $\quad 0$

B $\quad 1$

C $\quad 2$

D $\quad 3$

E $\quad 4$



☆ A against B. $A \uparrow \longrightarrow B$. y against a.

EASY.

9.

14. a, b, x, and y are real and positive.

a and b are constants.

x and y are related.

A graph of $\log y$ against $\log x$ is drawn.

For which one of the following relationships will this graph be a straight line?

$\hookrightarrow \log y = m\log x + n \quad m \in \mathbb{R}, n \in \mathbb{R}$

$= m\log x + n\log_{10} 10$

$= \log x^m + \log 10^n$

$\log y = \log(\underbrace{10^n}_{\text{Constant}} \times x^m)$

$y = 10^n \times x^m = ☆ \times x^m \cong a \cdot x^b$ ☆, $m \in \mathbb{R}$

A $\quad y^b = a^x$

B $\quad y = ab^x$

C $\quad y^2 = a + x^b$

D $\quad y = ax^b$

E $\quad y^x = a^b$

✓ Typical Question where the habit of leaning towards graph shines☆

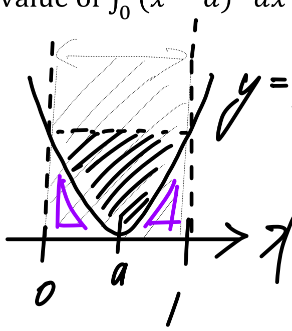
☆ Pure Graph Play.

$f(x) = ax^2 + \dots$ ↳ ☆ } area of quadratic $= \frac{|a|}{6}$ ☆^{3}

9.

15. The smallest possible value of $\int_0^1 (x-a)^2\,dx$ as a varies is

] Infer PDF of written solutions



$y = (x-a)^2$, so minimum at $a = \frac{1}{2}$.

$\rightarrow f(0) = \frac{1}{4}$.

$\frac{1}{4} \times 1 - \frac{1}{6} \times 1^3 = \frac{1}{12}$.

A $\quad \frac{1}{12}$

B $\quad \frac{1}{3}$

C $\quad \frac{1}{2}$

D $\quad \frac{7}{12}$

E $\quad 2$

* think of it as rectangle with base length of 1 moving in relation to $y = (x-a)^2$. Obviously

$\star$ 2 & 5 power play show up all the time.
Habituate prime factorisation.

EASY.

16. Given that c and d are non-zero integers, the expression $\dfrac{10^{c-2d} \times 20^{2c+d}}{8^{c} \times 125^{c+d}}$ is an integer if ______.

9.

A $c < 0$

B $d < 0$

C $c < 0$ and $d < 0$

D $c < 0$ and $d > 0$

E $c > 0$ and $d < 0$

F $c > 0$ and $d > 0$

G $d > 0$

H $c > 0$

$$\frac{(2\times5)^{c-2d} \times (2^2\times5)^{2c+d}}{(2^3)^c \times (5^3)^{c+d}}$$

$$= 2^{(c-2d)+2(2c+d)-(3c)} \times 5^{(c-2d)+(2c+d)-3(c+d)}$$

$$= \underset{\sim}{2}^{2c} \times 5^{-4d} \rightarrow \begin{cases} c > 0 \\ d < 0 \end{cases}$$

$\star\star\star$

$\star$ a little logic play here. in #16, it's given that d is integer. 5^{-4d} is an integer if BUT & d being a negative integer, is a SUFFICIENT but NOT NECESSARY condition for 5^{-4d} to be an integer.

why? try $-4d = \log_5 2$

$5^{\log_5 2} = 2^{\log_5 5} = 2.$

$a^{\log_b c} = c^{\log_b a}$

Just in case you did not know!

17. For what values of the non-zero real number a does the quadratic equation $ax^2 + (a-2)x = 2$ have real distinct roots?

9.

(1) a is variable of interest.

why I expanded: (2) I saw that I could factorise it

A All values of a

B $a = -2$

C $a > -2$

D $a \neq -2$

E No values of a

$ax^2 + ax = 2x + 2.$

$ax(x+1) = 2(x+1)$

$(ax-2)(x+1) = 0$

for this to not be $a(x+1)(x+1) = 0$,

$ax - 2 \neq ax + a$

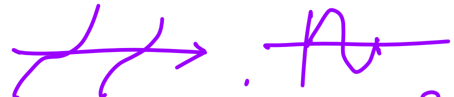
$\therefore \underline{a \neq -2}$

$\log_u c =$

a

9. Pour all terms having unknown variables to LHS. & try to factorise.

It was useless to sketch $y=\tan x$ itself like

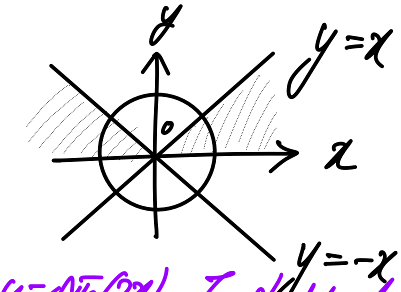


you get not much from it (you can do it this way, but why?)

9.

18. The angle x is measured in radians and is such that $0 \le x \le \pi$.

The total length of any intervals for which $-1 \le \tan x \le 1$ and $\sin 2x \ge 0.5$ is



at $x \in \{x \mid [0, \tfrac{1}{4}\pi] \cup [\tfrac{3}{4}\pi, \pi]\}$.

for $y=\tan(x)$, I sketched a unit circle.

for $y=\sin(2x)$, I sketched $y=\sin(2x)$ itself.

$\sin(2x) \ge \sin(\tfrac{1}{6}\pi)$: at $x \in [\tfrac{1}{12}\pi, \tfrac{5}{12}\pi]$.

A $\dfrac{\pi}{12}$

B $\dfrac{\pi}{6}$

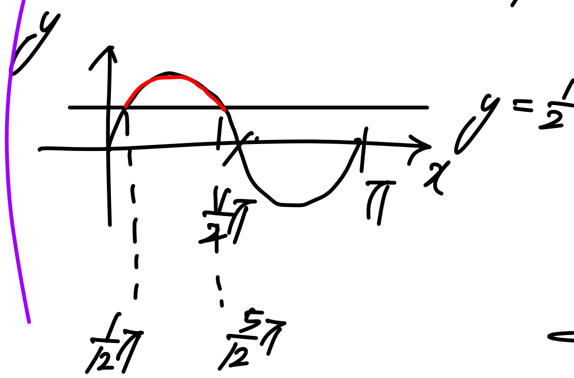
C $\dfrac{\pi}{4}$

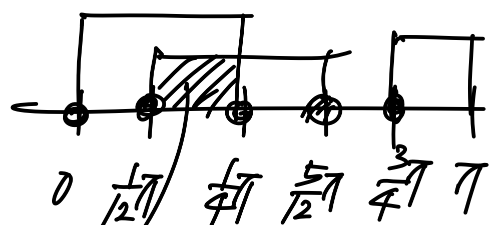
D $\dfrac{\pi}{3}$

E $\dfrac{5\pi}{12}$

F $\dfrac{\pi}{2}$

G $\dfrac{5\pi}{6}$





☆ for arithmetic sequence a_n: $a_{n-k}, a_n, a_{n+k} \rightarrow 2a_n = a_{n-k} + a_{n+k}$

length of interval: $\tfrac{1}{4}\pi - \tfrac{1}{12}\pi = \tfrac{1}{6}\pi$.

9.

19. A geometric series has first term 4 and common ratio r, where $0 < r < 1$. (so $r \ne 1$)

The first, second, and fourth terms of this geometric series form three successive terms of an arithmetic series.

The sum to infinity of the geometric series is

$4 \quad 4r \quad 4r^3$

$+☆ \quad +☆$

$r^2 + r - 1 = 0$

$r = \dfrac{-1+\sqrt{5}}{2}$ ($\because 0<r<1$)

$\dfrac{4}{1-r} = \dfrac{4}{1 - \frac{-1+\sqrt{5}}{2}} = 2(3+\sqrt{5})$

A $\frac{1}{2}(\sqrt{5} - 1)$

B $2(3 - \sqrt{5})$

C $2(1 + \sqrt{5})$

D $2(3 + \sqrt{5})$

sol 1)

$4r - 4 = 4r^3 - 4r$

$4(r-1) = 4r(r^2-1)$

$= 4r(r-1)(r+1)$

$r \ne 1$

$1 = r(r+1)$

sol 2) $2(4r) = 4 + 4r^3$

$2r = 1 + r^3$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -2 & 1 \\ & & 1 & 1 & -1 \\ \hline & 1 & 1 & -1 & 0 \end{array}$$

$(r-1)(r^2+r-1) = 0$.

☆ Mold. Morph, and Bend question into your favour.

9.

20. The coefficient of x^2 in the expansion of $(4 - x^2)[(1 + 2x + 3x^2)^6 - (1 + 4x^3)^5]$ is

A 28

B 72

C 78

D 192

E 240

F 310

G 312

(two avenues for term x^2 to show up

Current form: $(4-x^2) \times A(x)$.

① $4 \times$ (☆x^2 (☆ = real constant) term for $A(x)$)

② $(-x^2) \times$ (Constant term from $A(x)$)

Narrow down on aim:

Aim is to find two things: ((1) coefficient of x^2, (2) constant) at $A(x)$'s expansion.

END OF TEST

① x^2 coeff:

$A(x) = (1+2x+3x^2)^6 - (1+4x^3)^5$

all exponents of this one's expanded form are divisible by 3. so they have nothing to do with x^2 exponent.

$(3x^2)^1 (2x)^0 (1)^5 \times {}_6C_1 \times {}_5C_5$

$+ (3x^2)^0 (2x)^2 (1)^4 \times {}_6C_2 \times {}_4C_4$

$= 3 \times 6x^2 + 4 \times 15x^2 = \underline{78}x^2$.

② Constant at $A(x)$.

$(1)^6 (3x^2)^0 (2x)^0 \times {}_6C_6 - 1 = 0$.

$A(x) = 0 +$☆$x + 78x^2 + \cdots$

(in ascending order)

$(4-x^2)(0 + ?x + 78x^2 \cdots)$

$0 + ?x + (4 \times 78x^2 - 0x^2)$

9.

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