Model Answers Ideal Solution.

Cambridge Assessment Admissions Testing

TMUA/CTMUA Today's Colour: P!nk! **D513/01**

PAPER 1

November 2020 **75 minutes**

Additional materials: Answer sheet

6019449018

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one.
You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

This paper is the first of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 20 questions. Each question is worth one mark.

You can use the question paper for rough working or notes, but **no extra paper** is allowed.

You **must** complete the answer sheet within the time limit.

Calculators and dictionaries are NOT permitted.

There is no formulae booklet for this test.

Please wait to be told you may begin before turning this page.

This question paper consists of 21 printed pages and 3 blank pages.

PV2

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* Don't use Quotient Rule &
Simplify/Hold as fit prior to algebraic manipulation (integration/geometry)

1 Which of the following is an expression for the first derivative with respect to x of

$$\frac{x^3 - 5x^2}{2x\sqrt{x}}$$

A $-\frac{\sqrt{x}}{2}$

B $\frac{\sqrt{x}}{4}$

C $\frac{3x-5}{4\sqrt{x}}$

D $\frac{3\sqrt{x}-5}{4\sqrt{x}}$

E $\frac{3\sqrt{x}-10}{3\sqrt{x}}$

F $\frac{3x^2-10x}{3\sqrt{x}}$

$$f(x) = (2x\sqrt{x})^{-1}(x^3 - 5x^2)$$

$$= \frac{1}{2}x^{-\frac{3}{2}}(x^3 - 5x^2)$$

$$= \frac{1}{2}x^{\frac{3}{2}} - \frac{5}{2}x^{\frac{1}{2}}$$

$$\frac{d}{dx}f = \frac{3}{4}x^{\frac{1}{2}} - \frac{5}{4}x^{-\frac{1}{2}}$$

$$= \frac{1}{4}x^{-\frac{1}{2}}(3x-5)$$

$$= \frac{3x-5}{4\sqrt{x}}$$

☆ Remainder Theorem & Factor Theorem

2 $(2x+1)$ and $(x-2)$ are factors of $2x^3+px^2+q$

What is the value of $2p+q$?

A -10

B $-\frac{38}{5}$

C $-\frac{22}{3}$

D $\frac{22}{3}$

E $\frac{38}{5}$

F 10

let $f(x)=2x^3+px^2+q$,

$f(-\frac{1}{2})=2(-\frac{1}{2})^3+p(-\frac{1}{2})^2+q$

$=-\frac{1}{4}+\frac{1}{4}p+q=0$

$p+4q=1$

$f(2)=2\times2^3+p\times2^2+q$

$=16+4p+q=0$

$4p+q=-16$

substitute

$p=1-4q$: $4(1-4q)+q=-16$

$4-15q=-16$

$20=15q \rightarrow q=\frac{4}{3}$.

$p=1-4q=\frac{-13}{3}$

$2p+q=\frac{2(-13)+4}{3}$

$=\frac{-22}{3}$

3 Find the complete set of values of x for which

$$(x+4)(x+3)(1-x) > 0 \quad \text{and} \quad (x+2)(x-2) < 0$$

A $1 < x < 2$

B $-2 < x < 1$

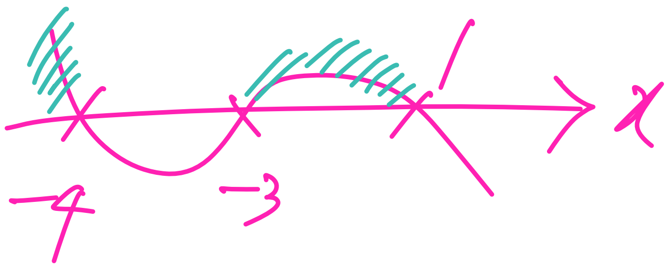
C $-2 < x < 2$

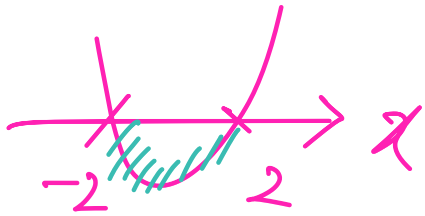
D $x < -2$ or $x > 1$

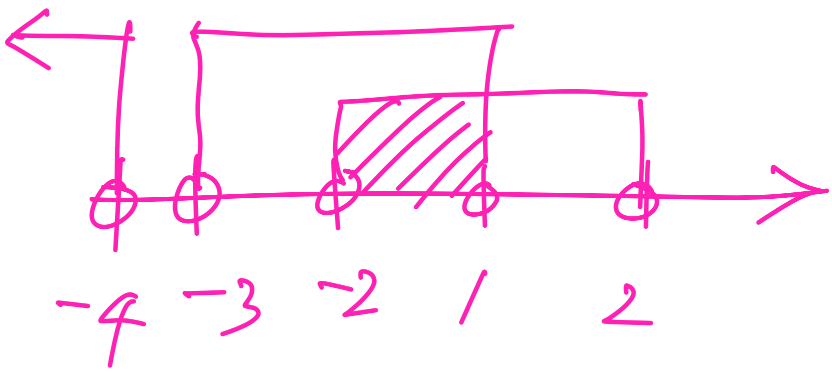
E $x < -4$ or $x > 2$

F $x < -4$ or $-3 < x < 1$

G $-4 < x < -2$ or $x > 1$







If you do $a \quad a+d \quad a+2d$ (which I know you did bro) Don't! you're wasting time & are more prone to error when you keep adding variables...

$a \quad ar^3 \quad ar^5$

$\star\ a_n^2 = a_{n-k}a_{n+k} \rightarrow$ (geometric)

$2b_n = b_{n-k} + b_{n+k}$ (Arithmetic)

4 The 1st, 2nd and 3rd terms of a geometric progression are also the 1st, 4th and 6th terms, respectively, of an arithmetic progression.

The sum to infinity of the geometric progression is 12.

Find the 1st term of the geometric progression.

A 1

B 2

C 3

D 4

E 5

F 6

two solutions.

① Structure with arithmetic & expand with geometric.

$\rightarrow \quad a \xrightarrow{\times r} a+3d \xrightarrow{\times r} a+5d$

$(a+3d)^2 = a(a+5d)$

$a^2 + 6ad + 9d^2 = a^2 + 5ad$

$ad + 9d^2 = 0$

$d(a+9d) = 0$

$a = -9d$

$r = \frac{a+3d}{a} = \frac{-6d}{-9d} = \frac{2}{3}$

$\frac{a}{1-\frac{2}{3}} = 12$

$a = 4$

② Vice versa.

$a \xrightarrow{+3d} ar \xrightarrow{+2d} ar^2$

$\rightarrow 6d = 2 \times 3d = 2 \times (ar - a)$

$= 3 \times 2d = 3 \times (ar^2 - ar)$

$2(ar-a) = 3r(ar-a)$

$ar \neq a$ ($\because r \neq 1$, because if it was then S_∞ diverges)

$r = \frac{2}{3}$

$\left(\frac{a}{1-\frac{2}{3}} = 12. \quad a = 4\right)$

5 The curve S has equation

$$y = px^2 + 6x - q$$

where p and q are constants.

S has a line of symmetry at $x = -\frac{1}{4}$ and touches the x-axis at exactly one point.

What is the value of $p + 8q$?

A 6

B 18

C 21

D 25

E 38

$$y = p\left(x + \frac{1}{4}\right)^2 + 0$$

$$= p\left(x^2 + \frac{1}{2}x + \frac{1}{16}\right)$$

$$= px^2 + \left(\frac{1}{2}p\right)x + \frac{1}{16}p$$

$$\frac{1}{2}p = 6, \quad p = 12$$

$$-q = \frac{1}{16}p = \frac{3}{4}$$

$$8q = -6.$$

$$12 + 8q = 6$$

6 Find the maximum value of the function

$$f(x) = \frac{1}{5^{2x} - 4(5^x) + 7}$$

A $\frac{1}{7}$

B $\frac{1}{4}$

C $\frac{1}{3}$

D 3

E 4

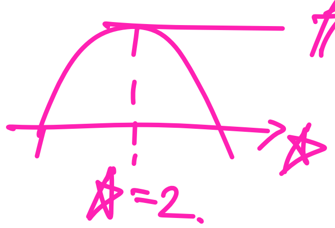
F 7

max(f(x)) = min(Denominator).

let $5^x = \star$ $(\star > 0)$.

$\star^2 - 4\star + 7$

$= (\star - 2)^2 + 3$.



max$((\star-2)^2+3)$ at $\star = 2$. $\rightarrow$ 3.

you should have still went over $\boxed{\star = 2}$ at 3 : if it was $(\star+2)^2+3$, then it would not be the case $(\because \star > 0)$.

7 Given that

$$2^{3x} = 8^{(y+3)}$$

$3x = 3(y+3)$

$x = y+3$

and

$$4^{(x+1)} = \frac{16^{(y+1)}}{8^{(y+3)}}$$

what is the value of $x + y$?

A -23

B -22

C -15

D -14

E -11

F -10

$$2^{2(x+1)} = \frac{2^{4y+4}}{2^{3(y+3)}}$$

$2x+2 = 4y+4-3y-9$

$2x+2 = y-5$

$y = 2x+7$

$y = 2(y+3)+7$

$y = -13$

$x = -10$

8 The function f is defined for all real x as

$$f(x) = (p - x)(x + 2)$$

Find the complete set of values of p for which the maximum value of $f(x)$ is less than 4.

A $-2 - 4\sqrt{2} < p < -2 + 4\sqrt{2}$

B $-2 - 2\sqrt{2} < p < -2 + 2\sqrt{2}$

C $-2\sqrt{5} < p < 2\sqrt{5}$

D $-6 < p < 2$

E $-4 < p < 0$

F $-2 < p < 2$

max(f(x)) at vertex,

$x = \frac{p-2}{2}$

$f\left(\frac{p-2}{2}\right) = \left(p - \frac{p-2}{2}\right)\left(\frac{p-2}{2} + 2\right)$

$= \frac{2p-p+2}{2} \times \frac{p-2+4}{2}$

$= \frac{(p+2)^2}{4}$

$\frac{(p+2)^2}{4} < 4$

$(p+2)^2 - 4^2 < 0$

$(p-2)(p+6) < 0$

$-6 < p < 2$

9 The quadratic expression $x^2 - 14x + 9$ factorises as $(x - \alpha)(x - \beta)$, where α and β are positive real numbers.

Which quadratic expression can be factorised as $(x - \sqrt{\alpha}\,)(x - \sqrt{\beta}\,)$?

A $x^2 - \sqrt{10}x + 3$

B $x^2 - \sqrt{14}x + 3$

C $x^2 - \sqrt{20}x + 3$

D $x^2 - 178x + 81$

E $x^2 - 176x + 81$

F $x^2 + 196x + 81$

$\alpha + \beta = 14$

$\alpha\beta = 9$

$\because$ Vieta's formulae.

$\sqrt{\alpha} + \sqrt{\beta} = ☆$

$(\quad)^2 = (\quad)^2$

$\alpha + \beta + 2\sqrt{\alpha\beta} = ☆^2$

LHS: $14 + 2\sqrt{9} = 20$. $☆ = \sqrt{20}$

$\sqrt{\alpha} \times \sqrt{\beta} = \sqrt{\alpha\beta} = 3$

$x^2 - \sqrt{20}x + 3$

10 The following sequence of transformations is applied to the curve $y = 4x^2$

1. Translation by $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$

2. Reflection in the x-axis

3. Stretch parallel to the x-axis with scale factor 2

What is the equation of the resulting curve?

A $y = -x^2 + 12x - 31$

B $y = -x^2 + 12x - 41$

C $y = \quad x^2 + 12x + 31$

D $y = \quad x^2 + 12x + 41$

E $y = -16x^2 + 48x - 31$

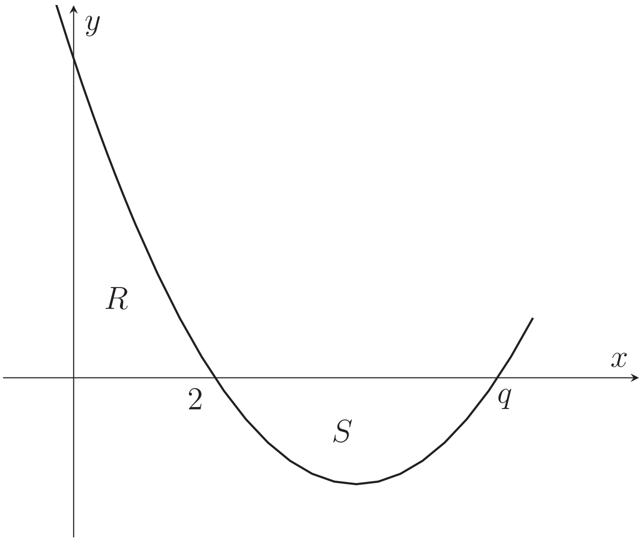
F $y = -16x^2 + 48x - 41$

G $y = \quad 16x^2 - 48x + 31$

H $y = \quad 16x^2 - 48x + 41$

$$f(x, y) = 0$$
$$\downarrow$$
$$f(x-3, y+5) = 0.$$
$$\downarrow$$
$$f(x-3, -y+5) = 0$$
$$\downarrow$$
$$f(\underline{\tfrac{1}{2}x - 3}, \underline{-y+5}) = 0.$$
$$(-y+5) = 4\left(\tfrac{1}{2}x - 3\right)^2$$
$$-y+5 = (x-6)^2$$
$$y = -(x-6)^2 + 5.$$
$$= -x^2 + 12x - 31$$

11 The quadratic function shown passes through $(2, 0)$ and $(q, 0)$, where $q > 2$.



What is the value of q such that the area of region R equals the area of region S ?

A $\sqrt{6}$

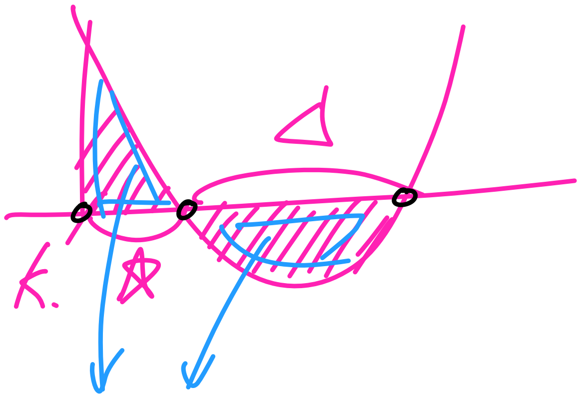
B 3

C $\frac{18}{5}$

D 4

E 6

F $\frac{33}{5}$



if shaded area is same, then
☆ : △ is 1 : 2.

☆ Sketch it & examine behaviour.

① Order in sketching graphs: Draw what you are more comfortable with fixating first.

12 How many real solutions are there to the equation

$$3\cos x = \sqrt{x}$$

where x is in radians?

A 0

B 1

C 2

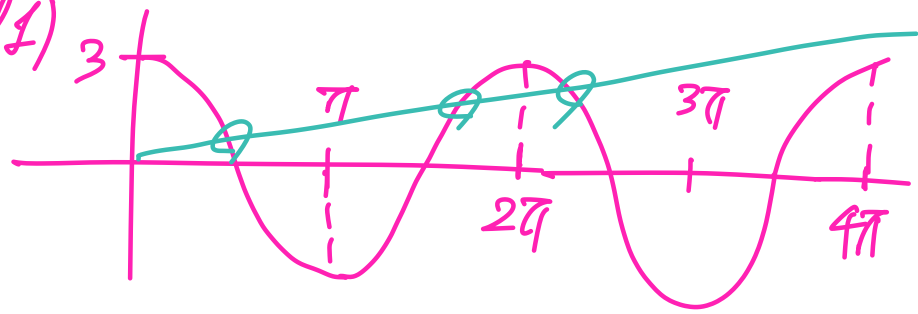
D 3

E 4

F 5

G infinitely many

sol(1)



$\sqrt{2\pi} = \underline{\sqrt{6.xx}} > \sqrt{4.xx}$

$\to 2.xx$

$\sqrt{4\pi} = \underline{\sqrt{12.xx}} > \sqrt{9.xx}$

$\to 3.xx$

Check analysis for algorithm behind this.

13 Find the coefficient of x^2y^4 in the expansion of $(1 + x + y^2)^7$

A 6

B 10

C 21

D 35

E 105

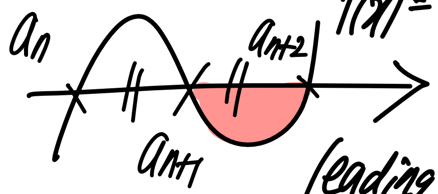
F 210

$$(1+x+y^2)^7$$

$$=(1)^{7-2-2}(x)^2(y^2)^2 \cdot \times {}_7C_2 \times {}_5C_2 \times {}_3C_3$$

$$=\frac{7\ 6}{2\ 1}\times\frac{5\ 4}{2\ 1}\times 1$$

$$=21\times 10 = 210.$$



$f(x)=a(x-a_n)(x-a_{n+1})(x-a_{n+2})$.

when $f(x)$ is a function such that leading coefficient $= a$, and 3 roots are equidistant (thus arithmetic sequence)

14 The area enclosed between the line $y = mx$ and the curve $y = x^3$ is 6.

What is the value of m?

A 2

B 4

C $\sqrt{3}$

D $\sqrt{6}$

E $2\sqrt{3}$

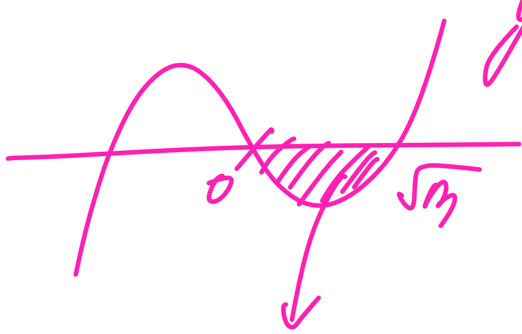
F $2\sqrt{6}$

dol Absolute G).

Shaded Area $= \frac{|a|}{4} d^4$.

Common difference of sequences.

$y = x^3 - mx$.



$\frac{1}{4}|1| \times \sqrt{m}^4 = 3$.

$\frac{1}{4} m^2 = 3$.

$m = \sqrt{12}$.

15 Find the positive difference between the two real values of x for which

$$(\log_2 x)^4 + 12\left(\log_2\left(\frac{1}{x}\right)\right)^2 - 2^6 = 0$$

A 4

B 16

C $\frac{15}{4}$

D $\frac{17}{4}$

E $\frac{255}{16}$

F $\frac{257}{16}$

let $\log_2 x = \bigstar$.

$\bigstar^4 + 12(-\bigstar)^2 - 2^6$

$= (\bigstar^2 + 16)(\bigstar^2 - 4)$

$\Rightarrow \bigstar^2 \geq 0$.

So $\bigstar^2 = 4$.

$\therefore \bigstar = \pm 2$.

$\log_2 x = 2, -2$

$x = 4, \frac{1}{4}$

★ I drew it completely different from how it actually is relative to the coordinates of x & y. → YOU DO NOT need to perfect it in relation to the coordinates. (far too many students obsess over Drawing it perfectly. Useless.)

16 The circle C_1 has equation $(x+2)^2 + (y-1)^2 = 3$

The circle C_2 has equation $(x-4)^2 + (y-1)^2 = 3$

The straight line l is a tangent to both C_1 and C_2 and has positive gradient.

The acute angle between l and the x-axis is θ

Find the value of $\tan\theta$

A $\frac{1}{2}$

B 2

C $\frac{\sqrt{2}}{2}$

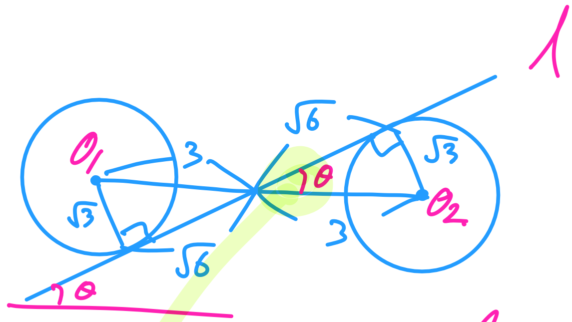
D $\sqrt{2}$

E $\frac{\sqrt{6}}{2}$

F $\frac{\sqrt{6}}{3}$

G $\frac{\sqrt{3}}{3}$

H $\sqrt{3}$



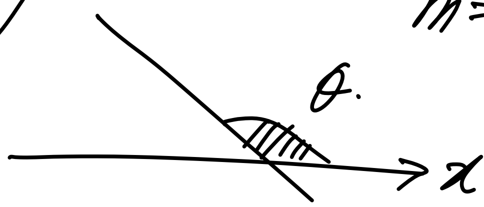
let O_1, O_2 be centre of C_1, C_2.

$\overline{O_1O_2}$ // x axis.

hence angle here is θ.

$\tan\theta = \frac{\sqrt{3}}{\sqrt{6}} = \frac{1}{\sqrt{2}}$.

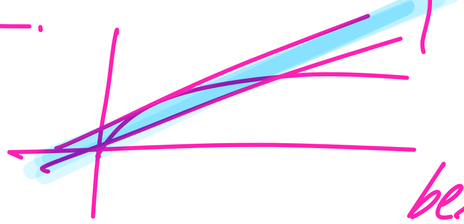
$y = mx + n$. (line)



$m = \tan\theta$, where θ is the angle formed anticlockwise from the $+x$ axis direction.

Just like Question 12. You have an obvious graph that you know you can draw.

17 Find the complete set of values of m in terms of c such that the graphs of $y = mx + c$ and $y = \sqrt{x}$ have two points of intersection.



* can only exist in between

A $0 < m < \frac{1}{4c}$

B $0 < m < 4c^2$

C $m > \frac{1}{4c}$

D $m < \frac{1}{4c}$

E $m > 4c^2$

F $m < 4c^2$

$mx+c=\sqrt{x}$

$m^2x^2+2mcx+c^2=x.$

$m^2x^2+(2mc-1)x+c^2=0.$

if $D>0$, they have 2 roots.

$D=(2mc-1)^2-4m^2c^2$

$=-4mc+1>0.$

$1>4mc.$

$\frac{1}{4c}>m$

Massive Caveat!!:

if $A=B$, then $A^2=B^2$.

But $A^2=B^2$ does not necessarily mean $A=B$. it could be $A=-B$.

So $(A=B) \rightarrow (A^2=B^2)$

$A=B$ is sufficient but not necessary for $(A^2=B^2)$.

Sum logic for y'all.

It has like 15+ solutions to solve this.

18. Find the number of solutions and the sum of the solutions of the equation

$$1 - 2\cos^2 x = |\cos x|$$

where $0 \leq x \leq 180°$

A Number of solutions $= 2$ Sum of solutions $= 180°$

B Number of solutions $= 2$ Sum of solutions $= 240°$

C Number of solutions $= 3$ Sum of solutions $= 180°$

D Number of solutions $= 3$ Sum of solutions $= 360°$

E Number of solutions $= 4$ Sum of solutions $= 240°$

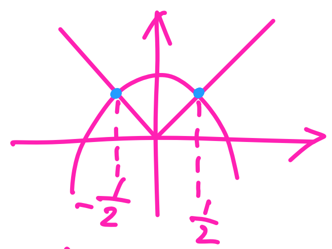
F Number of solutions $= 4$ Sum of solutions $= 360°$

sol 1). let $\cos x = \star$ (at $0° \leq \theta \leq 180°$; $-1 \leq \star \leq 1$)

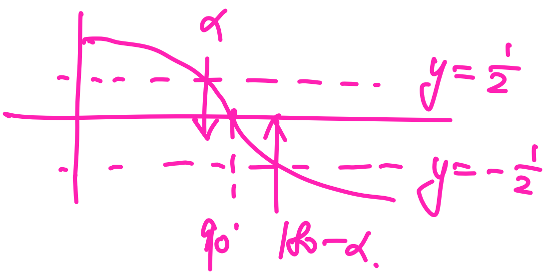
$1 - 2\star^2 = |\star|$

sol 1-1)

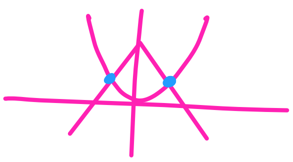
Directly: $1 - 2\star^2$ vs $|\star|$



$\star = \pm\frac{1}{2}$



(or) $1 - |\star|$ vs $2\star^2$



sol 1-2) Algebra

i) $\star \geq 0$.

$1 - 2\star^2 = \star$.

$2\star^2 + \star - 1 = 0$.

$(2\star - 1)(\star + 1) = 0$. $\star = \cancel{-1}, \frac{1}{2}$

ii) $\star < 0$

$1 - 2\star^2 = -\star$

$2\star^2 - \star - 1 = 0$.

$(2\star + 1)(\star - 1) = 0 \rightarrow \star = \cancel{1}, -\frac{1}{2}$

$\star = \pm\frac{1}{2}$.

Equations $\equiv$ Graphs.

(Roots) $\equiv$ intersections.

19 Find the lowest positive integer for which $\boxed{x^2 - 52x - 52 \text{ is positive.}}$

$\equiv x^2$ is above $52(x+1)$

$x^2 = 52(x+1).$

A 26

B 27

C 51

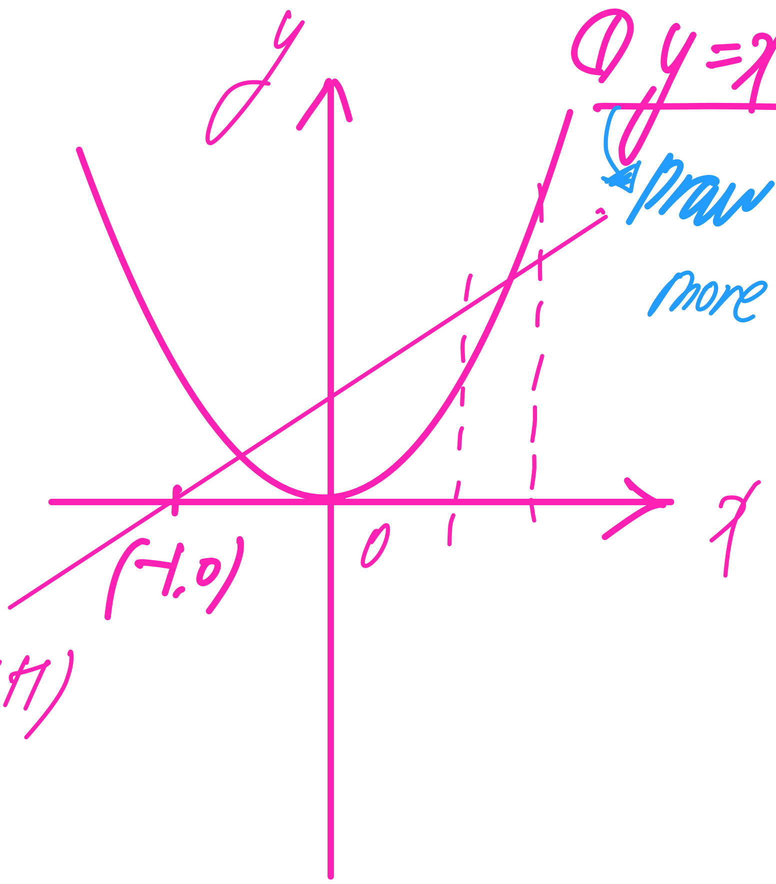
D 52

E 53

F 54

① $y = x^2$

Draw this first. more intuitive.



$y = 52(x+1)$

Just try stuff → at $x = 52$. $52^2 < 52 \times 53$

at $x = 53$. $53^2 > 52 \times 54$

$(53-1)(53+1)$

$= 53^2 - 1^2.$

20 For how many values of a is the equation

$$(x-a)(x^2-x+a)=0$$

satisfied by exactly two distinct values of x? ≡ one root is repeated ⊕ the other is distinct root.

A 0

B 1

C 2

D 3

E 4

F more than 4

(i) $x=a$ is repeated root.

$x-a$ is factor of $x^2-x+a=0$.

factor theorem

$a^2-a+a=0$

$a=0$.

(ii) $x^2-x+a=0$ has a repeated root α and $a \neq \alpha$

(if $a=\alpha$ that's $(x-a)(x^2-x+a)=(x-a)^3$ so only one root).

$D=0$. $a=\frac{1}{4}$.

Here $(x-a)(x^2-x+a)=(x-\frac{1}{4})(x-\frac{1}{2})^2$

so Distinct Roots.

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TMUA 9.

Jamie Baek.

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